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Wound Rotor Induction Generators (WRIGs): Steady State

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1.1 Introduction

Wound rotor induction generators (WRIGs) are provided with three phase windings on the rotor and on the stator. They may be supplied with energy at both rotor and stator terminals. This is why they are called doubly fed induction generators (DFIGs) or double output induction generators (DOIGs). Both motoring and generating operation modes are feasible, provided the power electronics converter that supplies the rotor circuits via slip-rings and brushes is capable of handling power in both directions.

As a generator, the WRIG provides constant (or controlled) voltage V_s and frequency f_1 power through the stator, while the rotor is supplied through a static power converter at variable voltage V_r and frequency f_2 . The rotor circuit may absorb or deliver electric power. As the number of poles of both stator and rotor windings is the same, at steady state, according to the frequency theorem, the speed ω_m is as follows:

$$\omega_m = \omega_1 \pm \omega_2; \quad \omega_m = \Omega_R \cdot p_1 \quad (1.1)$$

where

p_1 is the number of pole pairs

Ω_R is the mechanical rotor speed

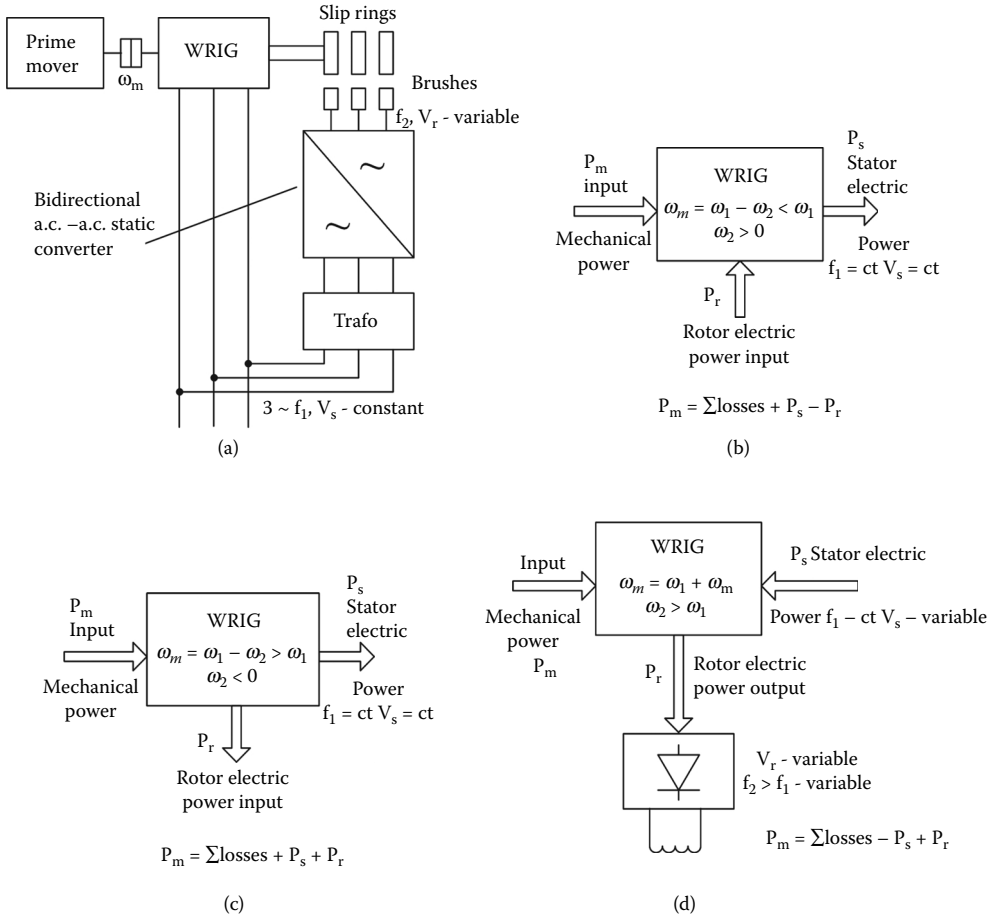


FIGURE 1.1 Wound rotor induction generator (WRIG) main operation modes: (a) basic configuration, (b) subsynchronous generating ($\omega_r < \omega_1$), (c) supersynchronous generating ($\omega_r > \omega_1$), and (d) rotor output WRIG (brushless exciter).

The sign is positive (+) in Equation 1.1 when the phase sequence in the rotor is the same as in the stator and $\omega_m < \omega_1$, that is, subsynchronous operation. The negative (−) sign in Equation 1.1 corresponds to an inverse phase sequence in the rotor when $\omega_m > \omega_1$, that is, supersynchronous operation.

For constant frequency output, the rotor frequency ω_2 has to be modified in step with the speed variation. This way, variable speed at constant frequency (and voltage) may be maintained by controlling the voltage, frequency, and phase sequence in the rotor circuit.

It may be argued that the WRIG works as a synchronous generator (SG) with three-phase alternating current (AC) excitation at slip (rotor) frequency $\omega_2 = \omega_1 - \omega_m$. However, as $\omega_1 \neq \omega_m$, the stator induces voltages in the rotor circuits even at steady state, which is not the case in conventional SGs. Additional power components thus occur.

The main operational modes of WRIG are depicted in Figure 1.1a through Figure 1.1d (basic configuration shown in Figure 1.1a). The first two modes (Figure 1.1b and Figure 1.1c) refer to the already defined subsynchronous and supersynchronous generations. For motoring, the reverse is true for the rotor circuit; also, the stator absorbs active power for motoring. The slip S is defined as follows:

$$S = \frac{\omega_2}{\omega_1} > 0; \text{ subsynchronous operation} \quad (1.2)$$

$$S = \frac{\omega_2}{\omega_1} < 0; \text{ supersynchronous operation}$$

A WRIG works, in general, for $\omega_2 \neq 0$ ($S \neq 0$), the machine retains the characteristics of an induction machine. The main output active power is delivered through the stator, but in supersynchronous operation, a good part, about slip stator powers (SPs), is delivered through the rotor circuit. With limited speed variation range, say from S_{max} to $-S_{max}$, the rotor-side static converter rating — for zero reactive power capability on the rotor side — would be $P_{conv} \approx |S_{max}| P_s$. With S_{max} typically equal to ± 0.2 to 0.25 , the static power converter ratings and costs would correspond to 20 to 25% of the stator delivered output power.

At maximum speed, the WRIG will deliver increased electric power, P_{max} :

$$P_{max} = P_s + P_{rmax} = P_s + |S_{max}| P_s \quad (1.3)$$

with the WRIG designed at P_s for $\omega_m = \omega_1$ speed. The increased power is delivered at higher than rated speed:

$$\omega_{mmax} = \omega_1 (1 + |S_{max}|) \quad (1.4)$$

Consequently, the WRIG is designed electrically for P_s at $\omega_m = \omega_1$, but mechanically at ω_{mmax} and P_{max} .

The capability of a WRIG to deliver power at variable speed but at constant voltage and frequency represents an asset in providing more flexibility in power conversion and also better stability in frequency and voltage control in the power systems to which such generators are connected.

The reactive power delivery by WRIG depends heavily on the capacity of the rotor-side converter to provide it. When the converter works at unity power delivered on the source side, the reactive power in the machine has to come from the rotor-side converter. However, such a capability is paid for by the increased ratings of the rotor-side converter. As this means increased converter costs, in general, the WRIG is adequate for working at unity power factor at full load on the stator side.

Large reactive power releases to the power system are still to be provided by existing SGs or from WRIGs working at synchronism ($S = 0$, $\omega_2 = 0$) with the back-to-back pulse-width modulated (PWM) voltage converters connected to the rotor controlled adequately for the scope.

Wind and small hydroenergy conversion in units of 1 megawatt (MW) and more per unit require variable speed to tap the maximum of energy reserves and to improve efficiency and stability limits. High-power units in pump-storage hydro- (400 MW [1]) and even thermopower plants with WRIGs provide for extra flexibility for the ever-more stressed distributed power systems of the near future. Even existing (old) SGs may be retrofitted into WRIGs by changing the rotor and its static power converter control.

The WRIGs may also be used to generate power solely on the rotor side for rectifier loads (Figure 1.1d). To control the direct voltage (or direct current [DC]) in the load, the stator voltage is controlled, at constant frequency ω_1 , by a low-cost alternating current (AC) three-phase voltage changer. As the speed increases, the stator voltage has to be reduced to keep constant the current in the DC load connected to the rotor ($\omega_2 = \omega_1 + \omega_m$). If the machine has a large number of poles ($2p_1 = 6, 8, 12$), the stator AC excitation input power becomes rather low, as most of the output electric power comes from the shaft (through motion).

Such a configuration is adequate for brushless exciters needed for synchronous motors (SMs) or for generators, where field current is needed from zero speed, that is, when full-power converters are used in the stator of the respective SMs or SGs.

With $2p_1 = 8$, $n = 1500$ rpm, and $f_1 = 50$ Hz, the frequency of the rotor output $f_2 = f_1 + n p_1 = 50 + (1500/60) \cdot 4 = 150$ Hz. Such a frequency is practical with standard iron core laminations and reduces the contents in harmonics of the output rectified load current.

In this chapter, the following subjects related to WRIG steady state will be detailed:

- Construction elements
- Basic principles
- Inductances
- Steady-state model (equations, phasor diagram, equivalent circuits)

- Steady-state characteristics at power grid
- Steady-state characteristics for isolated loads
- Losses and efficiency

1.2 Construction Elements

The WRIG topology contains the following main parts:

- Stator laminated core with N_s uniformly distributed slots
- Rotor laminated core with N_r uniformly distributed slots
- Stator three-phase winding placed in insulated slots
- Rotor shaft
- Stator frame with bearings
- Rotor copper slip-rings and stator (placed) brushes to transfer power to (from) rotor windings
- Cooling system

1.2.1 Magnetic Cores

The stator and rotor cores are made of thin (typically 0.5 mm) nonoriented grain silicon steel lamination provided with uniform slots through stamping (Figure 1.2.a). To keep the airgap reasonably small, without incurring large core surface harmonics eddy current losses, only the slots on one side may be open. On the other side of the airgap, they should be half closed or half open (Figure 1.2b).

Though, in general, the use of radial–axial ventilation systems led to the presence of radial channels between 60 and 100 mm long elementary stacks, at least for powers up to 2 to 3 MW, axial ventilation with single lamination stacks is feasible (Figure 1.3a and Figure 1.3b). As the airgap is slightly increased in comparison with standard induction motors, the axial airflow through the airgap is further facilitated. The axial channels (Figure 1.3a) in the stator and rotor yokes (behind the slot region) play a key role in cooling the stator and the rotor, as do the radial channels (Figure 1.3b) for the radial–axial ventilation.

The radial channels, however, are less efficient, as they are “traveled” by the windings, and thus, additional phase resistance and leakage inductance are added by the winding zones in the radial channel contributions. In very large, or long, stack machines, radial–axial cooling may be inevitable, but, as explained before, below 3 MW, the axial cooling in unistack cores, already in industrial use for induction motors, seems to be the way of the future.

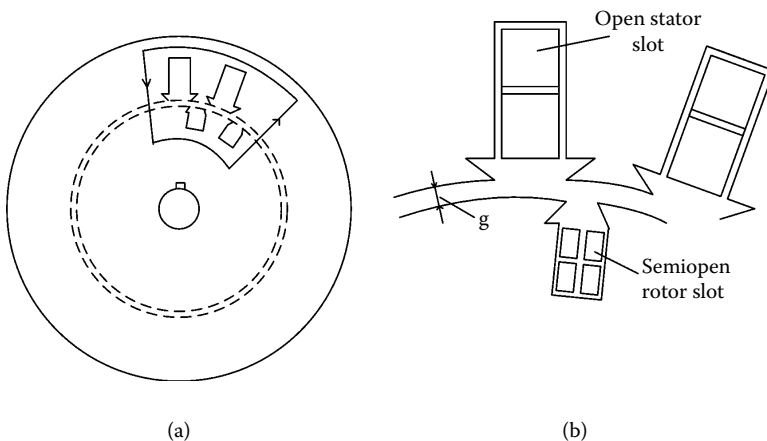


FIGURE 1.2 (a) Stator and (b) rotor slotted lamination.

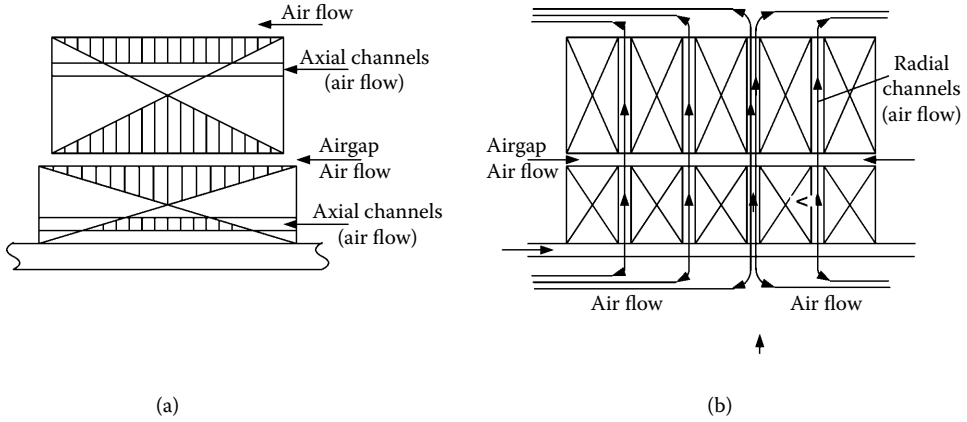


FIGURE 1.3 Stator and rotor stacks: (a) for axial cooling and (b) for radial-axial cooling.

1.2.2 Windings and Their mmfs

The stator and rotor three-phase windings are similar in principle. In Chapter 4 in *Synchronous Generators*, their design is described in some detail. Here, only the basic issues are presented. The three-phase windings are built to provide for traveling magnetomotive forces (mmfs) capable of producing a traveling magnetic field in the uniform airgap (slot openings are neglected or considered through the Carter coefficient $K_C = 1.02$ to 1.5):

$$B_g(x, t) = \frac{\mu_o F_{s,r}(x, t)}{gK_C(1 + K_s)} \quad (1.5)$$

where

$F_{s,r}(x, t)$ is equal to the mmfs per pole produced by either stator or rotor windings

g is the airgap

K_C is the Carter coefficient to account for airgap increase due to slot openings

K_s is the iron core contribution to equivalent magnetic reluctance of the main flux path (Figure 1.2a)

To produce a traveling airgap field, the stator and rotor mmfs, seen from the stator and from the rotor, respectively, have to be as follows:

$$F_s(\theta_s, t) = F_{1s} \cos(p_1 \theta_s - \omega_1 t) \quad (1.6)$$

$$F_r(\theta_r, t) = F_{1r} \cos(p_1 \theta_r \pm \omega_2 t) \quad (1.7)$$

where p_1 is the number of electrical periods of the magnetic field wave in the airgap or of pole pairs. The rotor mmf is produced by currents of frequency ω_2 .

At constant speed, the rotor and stator geometrical angles are related by

$$p_1 \theta_r = p_1 \theta_s - \omega_r t + \gamma; \quad \omega_r = \Omega_r \cdot p_1; \quad p_1 \theta_s = \omega_1 t \quad (1.8)$$

where ω_r is the rotor speed in electrical radians per second (rad/sec). Consequently, $F_r(\theta_s, t)$ becomes

$$F_r(\theta_s, t) = F_{1r} \cos[p_1 \theta_s - (\omega_r \pm \omega_2)t + \gamma] \quad (1.9)$$

The average electromagnetic torque and power per electric period is nonzero only if the two mmfs are at standstill with each other. That is,

$$\omega_1 = \omega_r \pm \omega_2; S = \omega_2 / \omega_1 \quad (1.10)$$

The positive sign (+) is used when $\omega_r < \omega_1$, and thus, the rotor and stator mmf waves rotate in a positive direction. The negative sign (−), used when $\omega_r > \omega_1$, refers to the case when the rotor mmf wave moves in the opposite direction to that of the stator. Also, the torque is nonzero when the angle $\gamma \neq 0$, that is, when the two mmfs are phase shifted.

To produce a traveling mmf, three phases, space lagged by 120° (electrical), have to be supplied by AC currents with 120° (electrical) time-lag angles between them (see Chapter 4 in *Synchronous Generators*, on the SG).

So, all three phase windings for, say, maximum value of current, should independently produce a sinusoidal spatial mmf:

$$(F_{sA,B,C}(\theta_s, t))_{t=0} = \frac{2}{3} F_{1s} \cos\left(p_1 \theta_s - (i-1) \frac{2\pi}{3}\right) \quad (1.11)$$

Each phase mmf has to produce $2p_1$ semiperiods along a mechanical period. With only one coil per pole per phase, there would be $2p_1$ coils per phase and $2p_1$ slots per phase if each coil occupies half of the slot (Figure 1.4a).

From the rectangular distribution of phase mmf (Figure 1.3a and Figure 1.3b), a fundamental is extracted:

$$F_{sA}(p_1, \theta_s) = \frac{4}{\pi} n_c I \sqrt{2} \cos p_1 \theta_s; \quad n_s - \text{turns/coil} \quad (1.12)$$

The harmonics content of the phase mmf in Figure 1.4b is hardly acceptable, but more steps in its distribution (more slots) and chorded coil would drastically reduce these space harmonics (Figure 1.5).

For the two-pole 24-slot winding with chorded coils (coil span/pole pitch = 10/12), the number of steps in the phase mmf is larger, and thus, the harmonics are reduced (Figure 1.5). For the fundamental component (based on Figure 1.5b), we obtain the expression of the mmf per pole and phase:

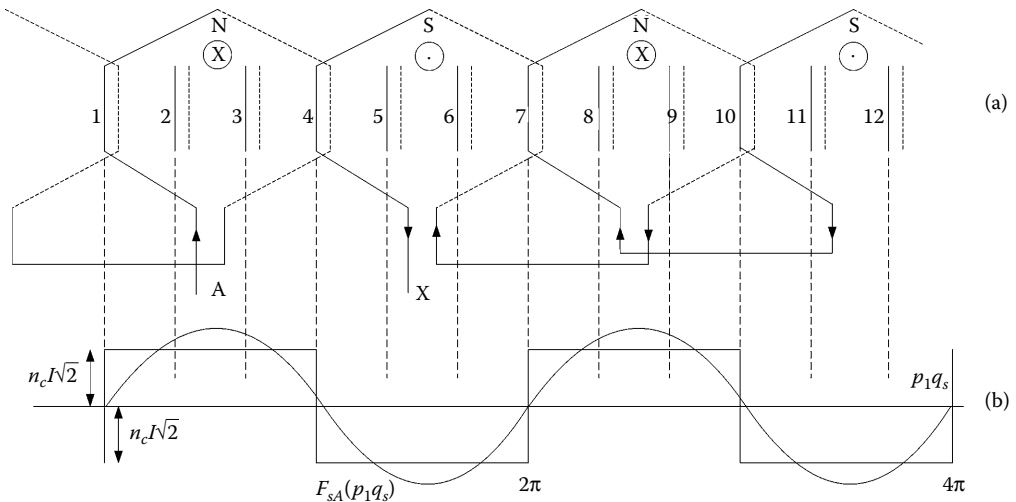


FIGURE 1.4 Elementary three-phase winding with $2p_1 = 4$ poles and $N_s = 12$ slots: (a) coils of phase A in series and (b) phase A magnetomotive force (mmf) for maximum phase current.

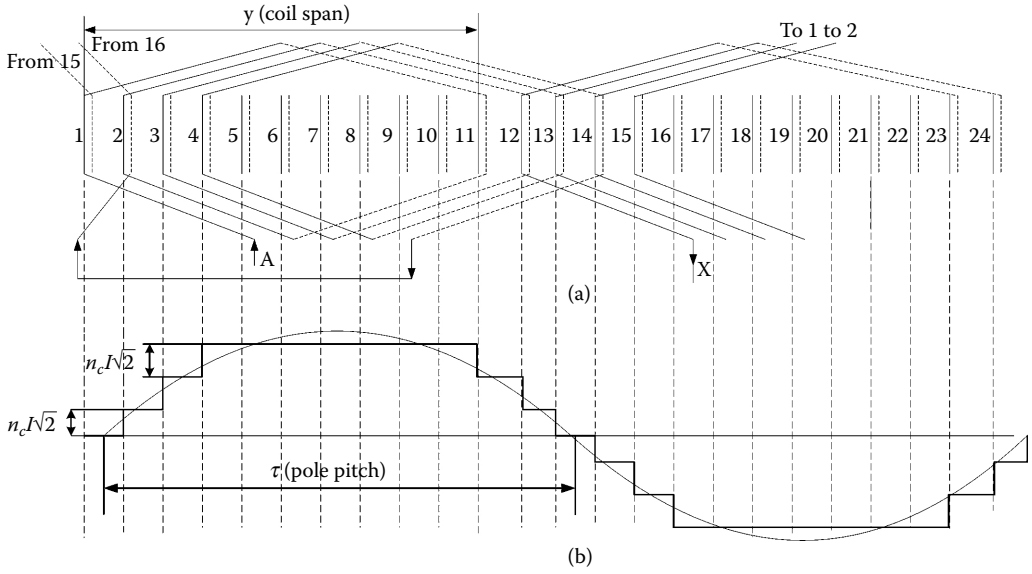


FIGURE 1.5 Two-pole ($2p_1 = 2$), $N_s = 24$ slots three-phase winding, with two layers in slot, coil span $y/\tau = 10/12$: (a) slot-to-phase allocation for layer 1 and coils of phase A and (b) phase A magnetomotive force (mmf) for maximum current.

$$F_{bA1} = \frac{2W_1 k_{w1} K_{y1} \cdot I\sqrt{2}}{\pi p_1}; \quad W_1 - \text{turns/phase} \quad (1.13)$$

For the space harmonic v , in a similar way,

$$F_{sAv} = \frac{2W_1 k_{dv} K_{yv} \cdot I\sqrt{2}}{vp_1} \quad (1.14)$$

with K_{dv} and K_{yv} known as distribution and chording factors:

$$K_{dv} = \frac{\sin v\pi/6}{q \sin \frac{v\pi}{6q}} \quad K_{yv} = \sin \frac{vy}{\tau} \frac{\pi}{2} \quad (1.15)$$

where q is the number of slots per pole per phase:

$$q_{s,r} = \frac{N_{s,r}}{2p_1 m_1} = \frac{N_{s,r}}{6p_1} \quad (1.16)$$

Only the odd harmonics are present, in general, as the positive and negative mmf poles are identical, while the multiples of three harmonics are zero for symmetric currents (equal amplitude, 120° phase shift): $v = 1, 5, 7, 11, 13, 17, 19, \dots$ It was proven (Chapter 4, in *Synchronous Generators*) that harmonics 7, 13, 19 are positive, and 5, 11, 17, $\dots$ are negative in terms of sequence. By adding the contributions of the three phases, we find that the mmf amplitude per pole F_{sv} is as follows:

$$F_{sv} = \frac{3}{2} F_{sAv} = \frac{3W_1 K_{dv} K_{yv} I\sqrt{2}}{vp_1} \quad (1.17)$$

Similar expressions may be derived for the rotor. To avoid parasitic synchronous torques, the number of slots of the stator and the rotor has to differ:

$$N_s \neq N_r; \quad q_s \neq q_r \quad (1.18)$$

Harmonics have to be treated carefully, as the radial magnetic pull due to rotor excentricity tends to be larger in WRIG than in cage-rotor induction generators (IGs) [2].

In general, WRIGs tend to be built with integer q both in the stator and in the rotor. Also, current paths in parallel may be used to reduce elementary conductor cross-sections.

Frequency (skin) effects have to be reduced, especially in large WRIGs, with bar-made windings where transposition may be necessary (Roebel bar, see Chapter 7, in *Synchronous Generators*).

Finally, the rotor winding end connections have to be protected against centrifugal forces through adequate bandages, as for cylindrical rotor SGs.

Whenever possible, the rated (design) voltage of the rotor winding has to be equal to that in the stator as required in the control of the rotor-side static power converter at maximum slip. This way, a voltage-matching transformer is avoided on the supply side of the static converter. Consequently, the rotor-to-stator turns ratio a_{rs} is as follows:

$$a_{rs} \cong \frac{W_r K_{q1}^r K_{d1}^r}{W_s K_{q1}^s K_{d1}^s} \cong \frac{1}{|S_{\max}|} \quad (1.19)$$

Care must be exercised in such designs to avoid connecting the stator at the full-voltage power grid at zero speed ($S = 1$), as the voltage induced in the rotor windings will be a_{rs} times larger than the rated one, jeopardizing the rotor winding insulation and the rotor-side static power converter.

If starting as a motor is required (for pump storage, etc.), it is done from the rotor, with the stator short-circuited, by making use of the rotor-side bidirectional power flow capabilities. Then, at certain speed $\omega_{rmin} > \omega_m(1 - |S_{\max}|)$, the stator circuit is opened. The machine is cruising while the control prepares the synchronization conditions by using the inverter on the rotor to produce adequate voltages in the stator. After synchronization, motoring (for pump storage) can be performed safely.

In WRIGs, a considerable amount of power (up to $|S_{\max}| \cdot P_{sN}$) is transferred in and out of the rotor electrically through slip-rings and brushes. With $|S_{\max}| = 0.20$, it is about 20% of the rated power of the machine. Remember that in SGs, the excitation power transfer to rotor by slip-rings and brushes is about five to ten times less.

The question is if those multimewatts may be transferred through slip-rings and brushes to the rotor in large-power WRIGs. The answer seems to be “yes,” as 200 MW and 400 MW units have been in operation for more than 5 years at up to 30 MW power transfer to the rotor.

In contrast to SGs, WRIGs have to use higher voltage for the power transfer to the rotor to reduce the slip-ring current. Multilevel voltage source bidirectional pulse-width modulated (PWM) MOSFET-controlled thyristor (MCT) converters are adequate for the scope of our discussion here. If the rotor voltage is increased in the kilovolt (and above) range, the insulation provisions for the rotor slip-rings and on the brush framing side are much more demanding.

Note that SG brushless exciters based on the WRIG principle with rotor rectified output do not need slip-rings and brushes. In WRIGs with large stator voltage ($V_n = 18$ kV, 400 MW), it may be more practical to use lower rated (maximum) voltage in the rotor, say up to 4.5 kV, and then use a step-up voltage adapting transformer to match the rotor connected static power converter voltage (4.5 kV) to the local (stator) voltage (say 18 kV). Such a reduction in voltage may reduce the eventual costs of the static power converter so much as to overcompensate the costs of the added transformer.

1.2.3 Slip-Rings and Brushes

A typical slip-ring rotor is shown in [Figure 1.6](#). It is obvious that three copper rings serve each phase, as the rotor currents are large.

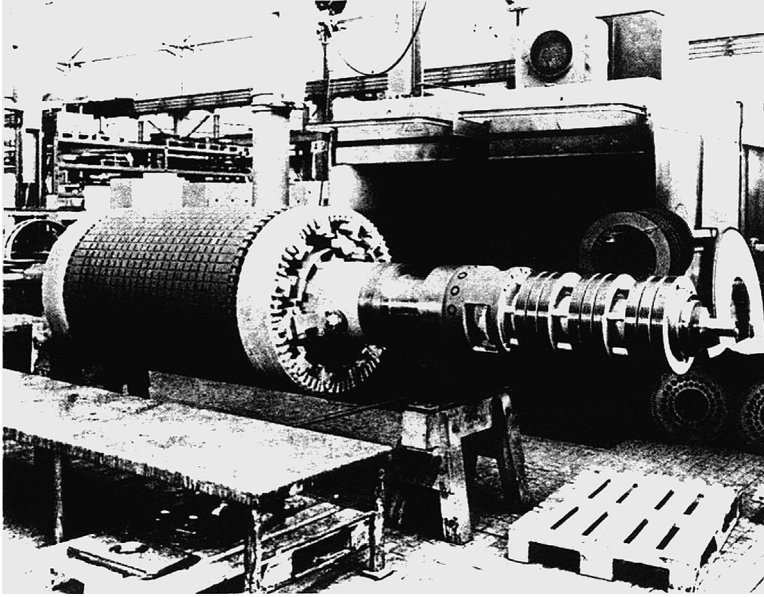


FIGURE 1.6 Slip-ring wound rotor.

1.3 Steady-State Equations

The electromagnetic force (emf) self-induced by the stator winding, with the rotor winding open, E_1 , is as follows:

$$E_1 = \pi \sqrt{2} f_1 W_1 K_{w1} \phi_{10}; \quad (RMS) \quad (1.20)$$

$$K_{w1} = K_{d1} \cdot K_{y1} \quad (1.21)$$

The flux per pole ϕ_{10} is

$$\phi_{10} = \frac{2}{\pi} B_{g10} \tau l_i \quad (1.22)$$

where

l_i is the stack length

τ is the pole pitch

D_{is} is the stator bore diameter

B_{g10} is the airgap fundamental flux density peak value:

$$B_{g10} = \frac{\mu_o F_{s10}}{K_c g (1 + K_s)} \quad (1.23)$$

F_{s10} is the amplitude of stator mmf fundamental per pole

From Equation 1.17, with $v = 1$,

$$F_{s10} = \frac{3 W_1 K_{w1} I_o \sqrt{2}}{\pi p_1} \quad (1.24)$$

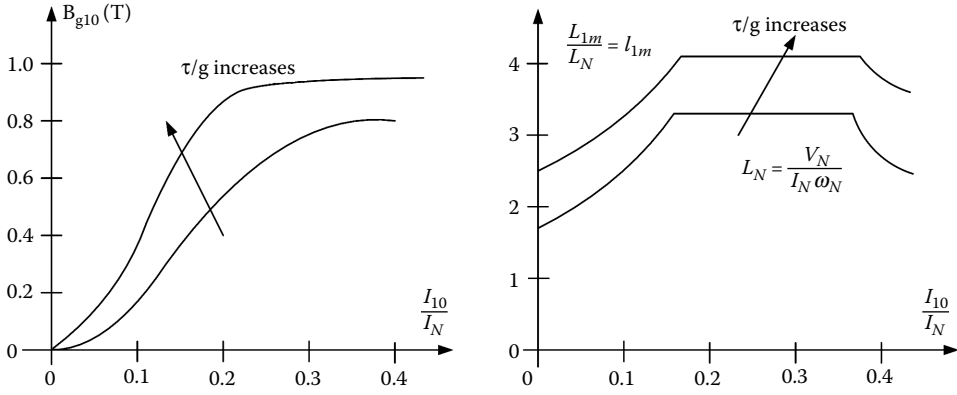


FIGURE 1.7 Typical airgap flux density (B_{g10}) and magnetization inductance (in per unit [P.U.]) vs. P.U. stator current.

But the same emf E_1 may be expressed as

$$E_1 = \omega_1 L_{1m} \cdot I_{10} \quad (1.25)$$

So, the main flux, magnetization (cyclic) inductance of the stator — with all three phases active and symmetric — L_{1m} is as follows (from Equation 1.20 through Equation 1.25):

$$L_{1m} = \frac{6\mu_0 (W_1 K_{W1})^2 \tau l_i}{\pi^2 p_1 K_C g (1 + K_s)} \quad (1.26)$$

The Carter coefficient $K_C > 1$ accounts for both stator and rotor slot openings ($K_C \approx K_{C1} K_{C2}$). The saturation factor K_s , which accounts for the iron core magnetic reluctance, varies with stator mmf (or current for a given machine), and so does magnetic inductance L_{1m} (Figure 1.7).

Besides L_{1m} , the stator is characterized by the phase resistance R_s and leakage inductance L_{sl} [2]. The same stator current induces an emf E_{2s} in the rotor open-circuit windings. With the rotor at speed ω_r — slip $S = (\omega_1 - \omega_r)/\omega_1$ — E_{2s} has the frequency $f_2 = Sf_1$:

$$\begin{aligned} E_{2s}(t) &= E_{2s} \sqrt{2} \cos \omega_2 t \\ E_{2s} &= \pi \sqrt{2} S f_1 W_2 K_{W2} \phi_{10} \end{aligned} \quad (1.27)$$

Consequently,

$$\frac{E_{2s}}{E_1} = S \frac{W_2 K_{W2}}{W_1 K_{W1}} = S \cdot K_{rs} \quad (1.28)$$

This rotor emf at frequency Sf_1 in the rotor circuit is characterized by phase resistance R_r^r and leakage inductance L_{rl}^r . Also, the rotor is supplied by a system of phase voltages at the same frequency ω_2 and at a prescribed phase.

The stator and rotor equations for steady-state/phase may be written in complex numbers at frequency ω_1 in the stator and ω_2 in the rotor:

$$(R_s + j\omega_1 L_{sl}) \underline{I}_s - \underline{V}_s = \underline{E}_1 \quad \text{at } \omega_1 \quad (1.29)$$

$$(R_r^r + jS\omega_1 L_{rl}^r) \underline{I}_r^r - \underline{V}_r^r = \underline{E}_{2s} \quad \text{at } \omega_2 \quad (1.30)$$

According to Equation 1.28, we may multiply Equation 1.30 by $1/K_{sr}$ to reduce the rotor to stator:

$$\begin{aligned} (R_r + jS\omega_1 L_{rl})\underline{I}_r - \underline{V}_r &= \frac{\underline{E}_{2s}}{K_{sr}}; \quad \underline{E}_{2s} = S\underline{E}_1 K_{sr} \\ R_r &= R_r^r / K_{rs}^2 \quad L_{rl} = L_{rl}^r / K_{rs}^2 \\ \underline{V}_r &= \underline{V}_r^r / K_{rs} \quad \underline{I}_r = \underline{I}_r^r \cdot K_{rs} \end{aligned} \quad (1.31)$$

The division of Equation 1.31 by slip S yields the following:

$$\left(\frac{R_r}{S} + j\omega_1 L_{rl} \right) \underline{I}_r - \frac{\underline{V}_r}{S} = \frac{S\underline{E}_1}{S} \quad (1.32)$$

But, Equation 1.31 may also be interpreted as being “converted” to frequency ω_1 , as $\underline{E}_1$ is at ω_1 ($\underline{E}_{2s}/S = \underline{E}_1$):

$$\left(\frac{R_r}{S} + j\omega_1 L_{rl} \right) \underline{I}_r - \frac{\underline{V}_r}{S} = \underline{E}_1; \quad \text{at } \omega_1 \quad (1.33)$$

In Equation 1.33, the rotor voltage $\underline{V}_r$ and current $\underline{I}_r$ vary with the frequency ω_1 and, thus, are written (in fact) in stator coordinates. A “rotation transformation” has been operated this way. Also, all variables are reduced to the stator. Physically, this would mean that Equation 1.33 refers to a rotor at standstill, which may produce or absorb active power to cover the losses and delivers in motoring the mechanical power of the actual machine it represents.

Finally, the emf $\underline{E}_1$ may now be conceived to be produced by both $\underline{I}_s$ and $\underline{I}_r$ (at the same frequency ω_1), both acting upon the magnetization inductance L_{1m} as the rotor circuit is reduced to the stator:

$$\underline{E}_1 = -j\omega_1 L_{1m} (\underline{I}_s + \underline{I}_r) = -j\omega_1 L_{1m} \underline{I}_m \quad (1.34)$$

1.4 Equivalent Circuit

The equivalent circuit corresponding to Equation 1.29, Equation 1.31, and Equation 1.34 is illustrated in Figure 1.8. Two remarks about Figure 1.8 are in order:

- The losses in the machine occur as stator and rotor winding losses $p_{cos} + p_{cor}$, core losses p_{Fe} , and mechanical losses p_{mec} :

$$p_{cos} = 3R_s I_s^2; \quad p_{cor} = 3R_r I_r^2; \quad p_{Fe} = 3R_{1m} (S\omega_1) I_{so}^2 \quad (1.35)$$

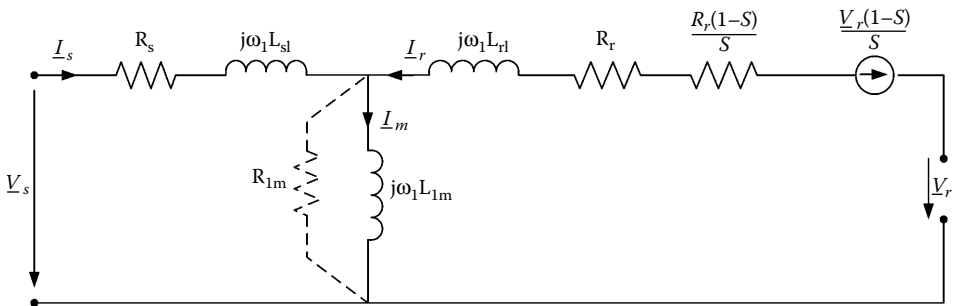


FIGURE 1.8 Wound rotor induction generator (WRIG) equivalent circuit for steady state.

- The resistance R_{lm} that represents the core losses depends slightly on slip frequency $\omega_2 = S\omega_1$, as non-negligible core losses also occur in the rotor core for $Sf_1 > 5$ Hz.
- The active power balance equations are straightforward, from Figure 1.8, as the difference between input electrical powers P_s and P_r and the losses represents the mechanical power P_m :

$$P_m = \left[3 \frac{R_r I_r^2}{S} - 3 \frac{\text{Re}(\underline{I}_r^* \underline{V}_r)}{S} \right] (1-S) = T_e \frac{\omega_1}{P_1} (1-S) = P_{elm} (1-S) \quad (1.36)$$

$$\Sigma p = p_{\cos} + p_{cor} + p_{mec} + P_{Fe}$$

P_{elm} is the electromagnetic (through airgap) power.

$$P_s + P_r^r = 3\text{Re}(\underline{V}_s \underline{I}_s^*) + 3\text{Re}(\underline{V}_r \underline{I}_r^*) = P_m + \Sigma p \quad (1.37)$$

T_e is the electromagnetic torque. The sign of mechanical power for given motion direction is used to discriminate between motoring and generating. The positive sign (+) of P_m is considered here for motoring (see the association of directions for $\underline{V}_s, \underline{I}_s$ in Figure 1.8).

The motor/generator operation mode is determined (Equation 1.36) by two factors: the sign of slip S and the sign and relative value of the active power input (or extracted) electrically from the rotor P_r (Table 1.1). So, the WRIG may operate as a generator or a motor both subsynchronously ($\omega_r < \omega_1$) and supersynchronously ($\omega_r > \omega_1$). The power signs in Table 1.1 may be portrayed as in Figure 1.9.

If all the losses are neglected, from Equation 1.36 and Equation 1.37:

$$P_m \approx -P_r \frac{(1-S)}{S} \approx P_s + P_r \quad (1.38)$$

Consequently,

$$P_r = -SP_s \quad (1.39)$$

The higher the slip, the larger the electric power absorption or delivery through the rotor. Also, it should be noted that in supersynchronous operation, both stator and rotor electric powers add up to convert the mechanical power. This way, up to a point, oversizing, in terms of torque capability, is not required when operation at $S = -S_{max}$ occurs with the machine delivering $P_s(1 + |S_{max}|)$ total electric power.

Reactive power flow is similar. From the equivalent circuit,

$$Q_s + Q_r = 3\text{Imag}(\underline{V}_s \underline{I}_s^*) + 3\text{Imag}\left(\frac{\underline{V}_r \underline{I}_r^*}{S}\right) = 3\omega_1 (L_{sl} I_s^2 + L_{rl} I_r^2 + L_m I_m^2) \quad (1.40)$$

TABLE 1.1 Operation Modes

S	0 < S < 1		S < 0	
	Subsynchronous ($\omega_r < \omega_1$)		Supersynchronous ($\omega_r > \omega_1$)	
Operation Mode	Motoring	Generating	Motoring	Generating
P_m	>0	<0	>0	<0
P_s	>0	<0	>0	<0
P_r	<0	>0	>0	<0

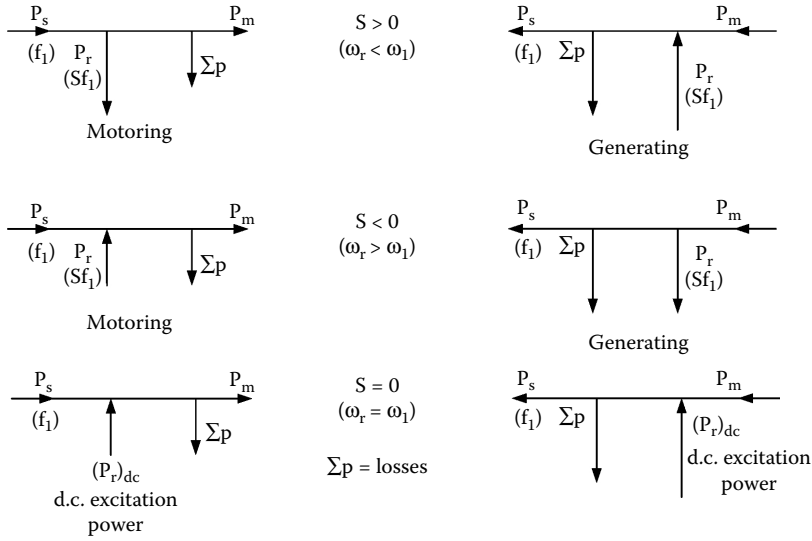


FIGURE 1.9 Operation modes of wound rotor induction generator (WRIG) at $S > 0$, $S < 0$, and $S = 0$.

So, the reactive power required to magnetize the machine may be delivered by the rotor or by the stator or by both. The presence of S in Equation 1.40 is justified by the fact that machine magnetization is perceived in the stator at stator frequency ω_1 .

As the static power converter rating depends on its rated apparent power rather than active power, it seems to be practical to magnetize the machine from the stator. In this case, however, the WRIG absorbs reactive power through the stator from the power grids or from a capacitive-resistive load. In stand-alone operation mode, however, the WRIG has to provide for the reactive power required by the load up to the rated lagging power factor conditions. If the stator operates at unity power factor, the rotor-side static power converter has to deliver reactive power extracted either from inside itself (from the capacitor in the DC link) or from the power grid that supplies it.

As magnetization is achieved with lowest kVAR in DC, when active power is not needed, the machine may be operated at synchronism ($\omega_r = \omega_1$) to fully contribute to the voltage stability and control in the power system. To further understand the active and reactive power flows in the WRIG, phasor diagrams are used.

1.5 Phasor Diagrams

To make better use of the phasor diagram, we will expose in the steady-state equations (Equation 1.29, Equation 1.33, and Equation 1.34) the phase flux linkages in the stator $\underline{\Psi}_s$, in the airgap $\underline{\Psi}_m$, and in the rotor $\underline{\Psi}_r$:

$$\begin{aligned}\underline{\Psi}_m &= L_{lm} \underline{I}_m; \quad \underline{I}_m = \underline{I}_s + \underline{I}_r \\ \underline{\Psi}_s &= \underline{\Psi}_m + L_{sl} \underline{I}_s; \quad \underline{\Psi}_s = L_s \underline{I}_s + L_{lm} \underline{I}_r; \quad L_s = L_{sl} + L_{lm} \\ \underline{\Psi}_r &= \underline{\Psi}_m + L_{rl} \underline{I}_r; \quad \underline{\Psi}_r = L_r \underline{I}_r + L_{lm} \underline{I}_s; \quad L_r = L_{rl} + L_{lm}\end{aligned}\tag{1.41}$$

All quantities in Equation 1.41 are reduced to the stator and “in-stator coordinates” — same frequency f_1 . With these new symbols, Equation 1.29, Equation 1.33, and Equation 1.34 become

$$\underline{I}_s R_s - \underline{V}_s = -j\omega_1 \underline{\Psi}_s; \quad \underline{I}_r R_r - \underline{V}_r = -j\omega_1 S \underline{\Psi}_r = +\underline{E}_r'\tag{1.42}$$

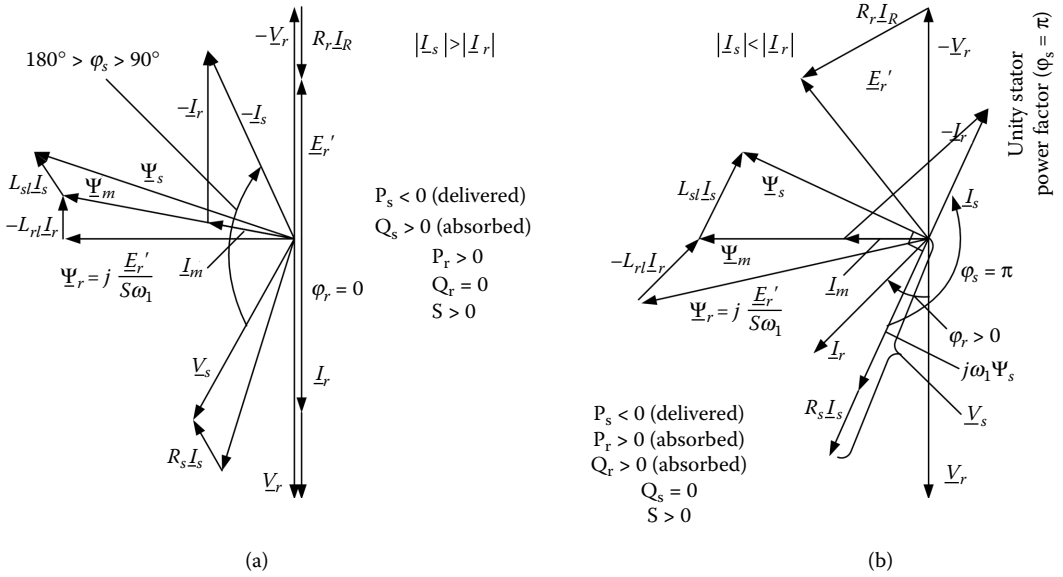


FIGURE 1.10 Phasor diagrams for wound rotor induction generator (WRIG) in generator mode, $S > 0$ ($\omega_r < \omega_l$): (a) for rotor unity power factor and (b) stator unity power factor.

To build the phasor diagrams, the value and sign of S and the phase shift ϕ_r between $\underline{V}_r$ and $\underline{I}_r$ in the rotor have to be known, together with machine parameters and the amplitude $|\underline{V}_r|$ of $\underline{V}_r$. Let us explore two cases: underexcitation and overexcitation, that is, respectively, with stator magnetization and rotor magnetization of the machine ($\cos \phi_s$ — leading and, respectively, lagging). For underexcitation conditions, we may assume unity power factor in the rotor ($\phi_r = 0$), as the magnetization is provided by the stator (Figure 1.10a), and start by drawing the $\underline{V}_r$ and $\underline{I}_r$ pair of phasors and then continue by using Equation 1.41 and Equation 1.42, alternatively, until $\underline{V}_s$ is obtained.

The phasor diagrams show that when the machine is underexcited, $\Psi_r < \Psi_s$ ($I_r < I_s$), while when it is overexcited, $\Psi_r > \Psi_s$ ($I_r > I_s$). The operation of WRIG may also be approached from the point of view of a synchronous machine.

From Equation 1.42,

$$\underline{I}_s(R_s + j\omega_l L_s) - \underline{V}_s = \underline{E}_p = -j\omega_l L_{lm} \underline{I}_r \quad (1.43)$$

Now, the problem is that the apparent synchronous reactance of the machine is L_s , the no-load inductance, while the emf $\underline{E}_p$ is produced only by the rotor current at stator frequency f_1 . As the slip $S \neq 0$, there is also interference between stator and rotor currents, so such an interpretation does not hold much promise in terms of practicality. However, the rotor flux $\underline{\Psi}_r$ in Equation 1.42 seems to be determined solely by the rotor voltage and current for given slip. To make use of this apparent decoupling, express $\underline{\Psi}_s$ as a function of $\underline{\Psi}_r$ and $\underline{I}_s$ from Equation 1.41:

$$\underline{\Psi}_s = \underline{\Psi}_r \cdot \frac{L_r}{L_{lm}} - L_{sc} \underline{I}_s; \quad L_r = L_{rl} + L_{lm}; \quad L_{sc} = L_s - \frac{L_{lm}^2}{L_r} \approx L_s + L_{rl} \quad (1.44)$$

Introducing Equation 1.44 in Equation 1.42 yields the following:

$$\underline{I}_s(R_s + j\omega_l L_{sc}) - \underline{V}_s = -j\omega_l \frac{L_r}{L_{lm}} \underline{\Psi}_r = \underline{E}_{\Psi_r}^s \quad (1.45)$$

Keeping the rotor flux constant, the machine behaves like a synchronous machine with synchronous reactance that is the short-circuit reactance X_{sc} . As $X_{sc} \ll X_s$, this “new machine” behaves much better in terms of stability and voltage regulation.

Controlling the WRIG to keep the rotor flux constant is practical and, in fact, it was extensively used in vector-controlled AC drives [3].

We may now totally eliminate $\underline{I}_r$ from Equation 1.42, with the following:

$$\underline{I}_r = \frac{\underline{\Psi}_r - L_{lm} \underline{I}_s}{L_r} \quad (1.46)$$

$$-R_r \frac{L_m}{L_r} \underline{I}_s + \left(\frac{R_r}{L_r} + jS\omega_1 \right) \underline{\Psi}_r = \underline{V}_r \quad (1.47)$$

$$(R_s + j\omega_1 L_{sc}) \underline{I}_s + j\omega_1 \frac{L_r}{L_m} \underline{\Psi}_r = \underline{V}_s \quad (1.48)$$

This set of equations is easy to solve, provided the stator voltage V_s , power P_s , and stator power factor angle φ_s are given:

$$\underline{I}_s = \frac{P_s}{3V_s \cos \varphi_s} \quad (1.49)$$

With V_s in the horizontal axis, the stator current phasor $\underline{I}_s$ is obtained:

$$\begin{aligned} \underline{V}_s &= V_s \\ \underline{I}_s &= I_s (\cos \varphi_s - j \sin \varphi_s) \end{aligned} \quad (1.50)$$

From Equation 1.48, $\underline{\Psi}_r$ is determined as amplitude and phase with respect to stator voltage. Then, rotor current $\underline{I}_r$ — in stator phase coordinates — can be computed from Equation 1.46, both in amplitude and phase. Finally, if the speed ω_r is known, the slip S is known ($S = 1 - \omega_r/\omega_1$) and, thus, from Equation 1.47, the required rotor voltage phasor $\underline{V}_r$ (in stator coordinates) is computed (V_r, δ_{Vr}).

Example 1.1

Consider a WRIG with the following data: $P_{SN} = 12.5$ MW, $\cos \varphi_N = 1$, $V_{SN1} = 6$ kV/(star connection) at $S_{max} = -0.25$, the turn ratio $K_{rs} = 1/S_{max} = 4.0$, $r_s = r_r = 0.0062$ (P.U.), $r_m = \infty$, $l_{s1} = l_{r1} = 0.0625$ (P.U.), $l_{lm} = 5.00$ (P.U.), $f_{1N} = 50$ Hz, $2p_1 = 4$ poles. Calculate:

- The parameters $R_s, R_r, X_{sl}, X_{rl}, X_{lm}$ in Ω
- For $S = -S_{max}$ and maximum power P_{max} at $\cos \varphi_s = 1$, calculate the rotor current, rotor voltage, and its angle δ_{Vr} with respect to the stator voltage, rotor active and reactive power P_r, Q_r , and total electric generator power $P_g = P_s + P_r$.

Solution

- The stator current at P_{SN} and $\cos \varphi_s = 1$ is

$$(I_s)_{S=-S_{max}} = \frac{P_{SN}}{\sqrt{3} V_{SN} \cos \varphi_N} = \frac{12.5 \times 10^6}{\sqrt{3} \cdot 6000 \cdot 1} = 1.204 \times 10^3 \text{ A}$$

Based on the definition of base reactance X_N , the latter is

$$X_N = \frac{V_{SN}/\sqrt{3}}{I_{SN}} = \frac{6000}{\sqrt{3} \cdot 1204} = 2.88 \, \Omega$$

$$R_s = R_r = r_s \cdot X_N = 0.00625 \cdot 2.88 = 0.018 \, \Omega$$

$$X_{sl} = X_{rl} = l_{sl} \cdot X_N = 0.0625 \cdot 2.88 = 0.18 \, \Omega$$

$$X_{lm} = l_{rm} \cdot X_N = 5 \times 2.88 = 14.4 \, \Omega$$

- The maximum current I_s is in phase opposition with the stator voltage as $\varphi_s = -180^\circ$ in the generator mode, and as in Equation 1.47 and Equation 1.48, absorbed powers are positive. The phasor diagram for this case is shown in Figure 1.11. From Equation 1.48, the rotor flux phasor $\underline{\Psi}_r$ is obtained:

$$\underline{\Psi}_r = -j(6000/\sqrt{3} - 1204 \cdot (0.018 + j \cdot 2 \cdot 0.18)) \cdot \frac{14.4}{2\pi \cdot 50} = 1.363 - j10.9756$$

The rotor current $\underline{I}_r$ is as follows (Equation 1.46):

$$\underline{I}_r = \frac{(1.363 - j10.9756) \cdot 314}{(0.18 + 14.4)} - \frac{14.4 \cdot (-1204)}{0.18 + 14.4} = 1218.49 - j236.3$$

From Equation 1.47, we can now compute the rotor voltage phasor $\underline{V}_r$ for $S_{max} = -0.25$:

$$\begin{aligned} \underline{V}_r &= -0.018 \cdot \left(\frac{14.4}{14.4 + 0.18} \right) (-1204) + \left[\frac{0.018 \cdot 314}{14.4 + 0.18} + j(-0.25) \cdot 314 \right] (1.363 - j10.9756) \\ &= -840 - j111.24 \end{aligned}$$

The reactive power through rotor Q_r , perceived at stator frequency, is (Figure 1.11)

$$Q_r = 3 \operatorname{Imag} \left(\frac{V_r I_r^*}{S} \right) = 3 \operatorname{Imag} \frac{(-840 - j111.14)(1218 + j236.3)}{-0.25} = 4.004 \, \text{MVAR}$$

In our case, $Q_s = 0$, so Q_r has to completely cover the reactive in the WRIG at stator frequency:

$$\begin{aligned} Q_r &= 3X_{ls} I_s^2 + 3X_{ls} I_r^2 + 3X_{lm} |I_s + I_r|^2 \\ &= 3 \times 0.018(1204^2 + 1218^2 + 236.3^2) + 3 \cdot 14.4 | -1204 + 1218 - j236.3 |^2 = 4.04 \, \text{MVAR} \end{aligned}$$

As expected, the two values of Q_r are very close to each other. Positive Q_r means absorbed reactive power, as it should, to fully magnetize the machine from the rotor ($Q_s = 0$). Q_r should not be confused with the reactive power Q_r^r that is measured at the slip-rings, at frequency $S\omega$:

$$Q_r^r = |S| Q_r = |0.25| \cdot 4.04 \, \text{MVA} = 1.01 \, \text{MVA}$$

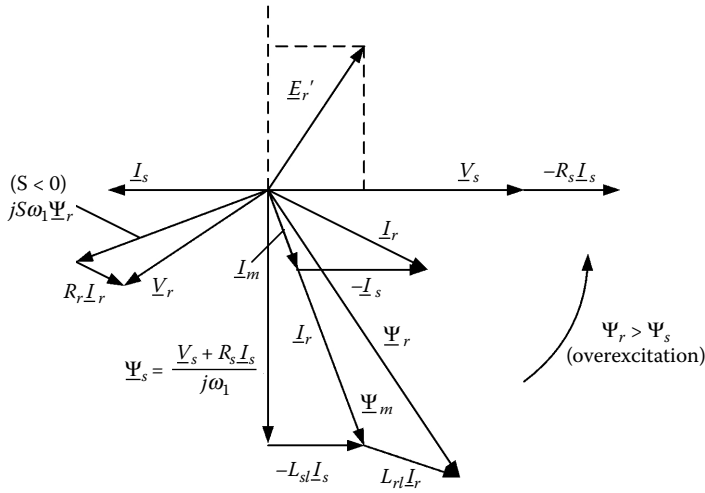


FIGURE 1.11 Phasor diagram for generating and unity stator power factor for $S < 0$.

The absolute value of slip is used to account for both subsynchronous and supersynchronous situations correctly, preserving the sign of the rotor-side reactive power.

The winding losses in the machine are the only losses considered in our example:

$$\begin{aligned}\Sigma P &= 3R_s I_s^2 + 3R_r I_r^2 = 3 \cdot 0.018 \cdot (1204^2 + 1218^2 + 236.3^2) \\ &= 161.404 \cdot 10^3 \text{ W} = 161.404 \text{ kW}\end{aligned}$$

The active power P_r^r through the rotor slip-rings is as follows:

$$P_r^r = 3 \operatorname{Re}(V_r I_r^*) = 3 \cdot (-840 - j111.24)(1218 + j2363) = -2.9905 \cdot 10^6 \text{ W} = -2.9905 \text{ MW}$$

The mechanical power (Equation 1.36) is as follows:

$$P_m = \left[3R_r I_r^2 - 3\operatorname{Re}(V_r I_r^*) \right] \left(\frac{1-S}{S} \right)$$

$$= \left[\frac{3 \cdot 0.018(1218^2 + 236^2)}{-0.25} + \frac{2.995 \cdot 10^6}{-0.25} \right] (1+0.25) = -15.39 \cdot 10^6 \text{ W} = -15.39 \text{ MW}$$

Checking the power balance Equation 1.37 shows the errors in our calculations (the losses in the machine are rather small at 1%):

$$P_s + P_r = -12.5 - 2.9905 = -15.4905 \text{ MW}$$

The mechanical power P_m absolute value should have been larger than $|P_i + P_r|$ by the losses in the machine. This is not the case, and care must be exercised when doing complex number calculations in order to be precise, especially for very high-efficiency machines. The computation of reactive power showed very good results because it has been rather large. Now, the megavoltampere (MVA)

rating of the rotor-side converter considered for $S_{max} = -0.25$ and unity power factor in the stator is as follows:

$$P_{ap}^r = \sqrt{P_r^{r2} + Q_r^{r2}} = \sqrt{2.9905^2 + 1.01^2} = 3.156 \text{ MVA} \quad (1.51)$$

The oversizing of the converter is not notable for unity power factor in the stator. With a turn ratio $a_{rs} = 4/1$, at $S_{max} = -0.25$, the rotor circuit will be fed at about the rated voltage of the stator and at rotor current reduced by a_{rs} time with respect to that calculated:

$$V_r^{real} = |\underline{V}_r| a_{rs} = \sqrt{840^2 + 111.24^2} \cdot 4 = 3,389 \text{ V} \quad (1.52)$$

$$I_r^{real} = |\underline{I}_r| / a_{rs} = \sqrt{1218^2 + 236.3^2} / 4 = 310 \text{ A} \quad (1.53)$$

It should also be noted that for overexcitation, when $\Psi_r > \Psi_s$ and $I_r = 1240.7 \text{ A} > I_s = 1204 \text{ A}$ and the WRIG is used in a configuration with a large number of poles, the magnetization reactance decreases (in P.U.) notably, and thus, the reactive power requirement from the rotor is larger. Consequently, the static power converter connected to the rotor should provide for it, directly, if the latter also works at the unity power factor at the source side. The back-to-back (bidirectional) PWM voltage source converter seems to be fully capable of providing for such requirements through the right sizing of the DC link capacitor bank.

1.6 Operation at the Power Grid

The connection of a WRIG to the power grid is similar to the case of an SG. There is, however, an exceptional difference: the rotor-side static converter provides the conditions of synchronization at any speed in the interval $\omega_r(1 \pm |S_{max}|)$ and electronically brings the stator open-circuit voltages at the same frequency and phase with the power grid. In fact, the control system (Chapter 2) has a sequence for synchronization. Always successful, synchronization is feasible in a short time, in contrast to SGs, for which frequency and phase may be adjusted only through refined speed control by the turbine governor that tends to be slow due to high mechanical inertia. Furthermore, the WRIG may be started as a motor with the stator short-circuited, and then, above $\omega_r(1 - |S_{max}|)$, the stator circuit is opened. Subsequently, the synchronization control may be triggered, and, after synchronization, the machine is loaded either as a motor ($P_s > 0$) or as a generator ($P_s < 0$) through adequate closed-loop fast control (Figure 1.12).

Once connected to the power grid, it is important to describe its active and reactive power capabilities at constant voltage and frequency ω_1 but at variable speed ω_r (and $\omega_2 = \omega_1 - \omega_r$).

To describe the operation at the power grid, the powers P_s and P_r vs. power angle, for given speed (slip) and rotor voltage are considered to be representative. To simplify the characteristics $P_s(\delta_{vr})$ and $P_r(\delta_{vr})$, the stator resistance is neglected. The power angle is taken as the angle between $\underline{V}_s$ and $\underline{V}_r$ (in stator coordinates) (Figure 1.13).

1.6.1 Stator Power vs. Power Angle

The machine steady-state Equation 1.41 and Equation 1.42 with currents $\underline{I}_s$ and $\underline{I}_r$ for $R_s = 0$ are as follows:

$$\underline{V}_s = j\omega_1(L_s \underline{I}_s + L_{lm} \underline{I}_r) \quad (1.54)$$

$$\underline{V}_r = R_r \underline{I}_r + jS\omega_1(L_r \underline{I}_r + L_{lm} \underline{I}_s) = V_r(\cos \delta + j \sin \delta) \quad (1.55)$$

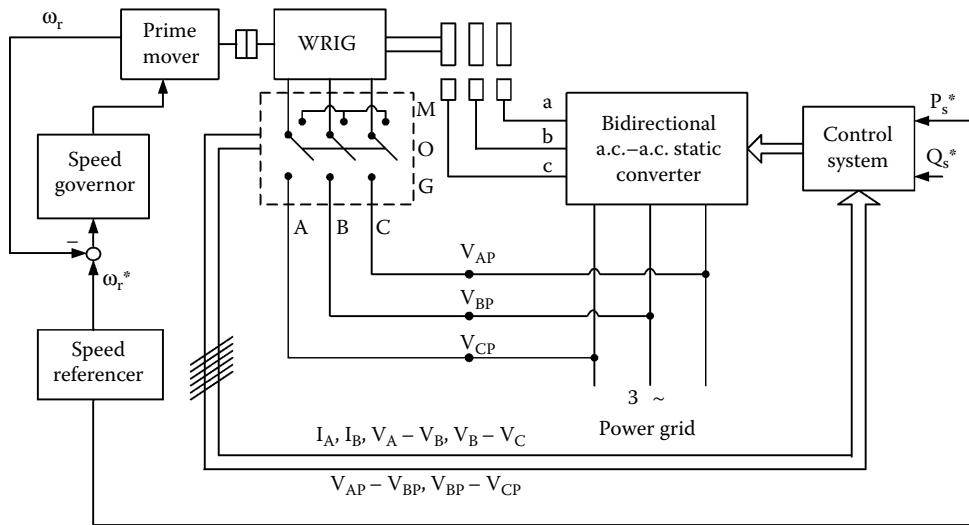


FIGURE 1.12 Synchronization arrangement for wound rotor induction generator (WRIG): M — motor starting, O — synchronization preparation mode, and G — generator at power grid.

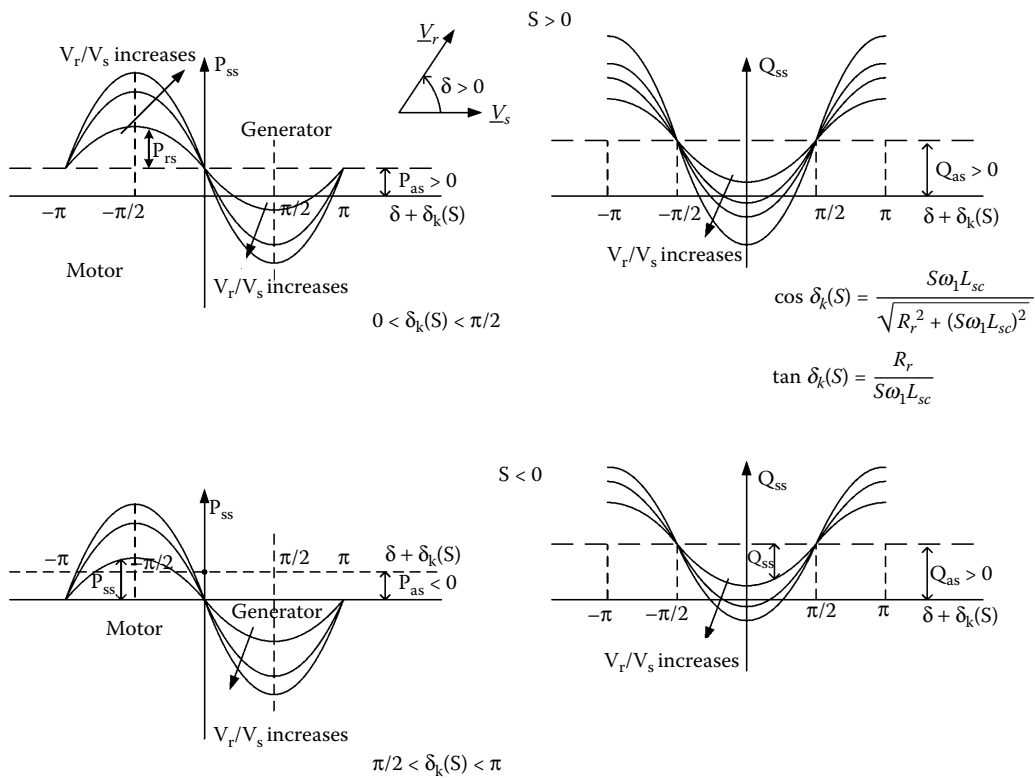


FIGURE 1.13 Powers P , Q vs. power angle $\delta + \delta_k(S)$.

Eliminating L_s from (1.54) yields the following:

$$R_r + \left(1 + jS\omega_1 \left(L_s - \frac{L_{1m}^2}{L_s} \right) \right) I_r = V_r (\cos \delta + j \sin \delta) - S V_s \frac{L_{1m}}{L_s} \quad (1.56)$$

With $L_r - L_{1m}^2/L_s \approx L_{sc}$,

$$I_r = \frac{\left(V_r \cos \delta - S V_s \frac{L_{1m}}{L_s} + j V_r \sin \delta \right) (R_r - j S \omega_1 L_{sc})}{R_r + (S \omega_1 L_{sc})^2} \quad (1.57)$$

The stator active and reactive powers P_s , Q_s from Equation 1.54 are

$$\begin{aligned} P_s + jQ_s &= 3V_s I_s^* = 3L_{1m} \frac{V_s}{L_s} \left(\frac{jV_s}{\omega_1 L_{1m}} - I_r^* \right) = 3 \frac{jV_s^2}{\omega_1 L_s} \\ &\quad - \frac{\left(V_r \cos \delta - S V_s \frac{L_{1m}}{L_s} - j V_r \sin \delta \right) (R_r + j S \omega_1 L_{sc})}{R_r^2 + S^2 \omega_1^2 L_{sc}^2} \cdot 3 \frac{L_{1m}}{L_s} V_s \end{aligned} \quad (1.58)$$

Expression 1.59 becomes

$$\begin{aligned} P_s &= -3V_s V_r \frac{L_{1m}}{L_s} \frac{\sin(\delta + \delta_k(S))}{\sqrt{R_r^2 + (S \omega_1 L_{sc})^2}} + 3V_s^2 \left(\frac{L_{1m}}{L_s} \right)^2 \frac{R_r S}{R_r^2 + (S \omega_1 L_{sc})^2} \\ &\quad \text{synchronous active power} \quad \text{asynchronous active power} \\ &\quad (P_{ss}) \quad (P_{as}) \end{aligned} \quad (1.59)$$

$$\begin{aligned} Q_s &= \frac{3V_s^2}{\omega_1 L_s} \left[1 + \frac{(S \omega_1 L_{1m})^2 L_{sc}}{[R_r^2 + (S \omega_1 L_{sc})^2] \cdot L_s} \right] - 3V_s V_r \frac{L_{1m}}{L_s} \frac{\cos(\delta + \delta_k(S))}{\sqrt{R_r^2 + (S \omega_1 L_{sc})^2}} \\ &\quad \text{asorbed reactive power} \quad \text{synchronous reactive power } (Q_{ss}) \\ &\quad \text{with short-circuited rotor } (Q_{as}) \end{aligned} \quad (1.60)$$

The resemblance to the nonsalient-pole SG is evident. However, the second term in P_s is produced asynchronously and is positive (motoring) for positive slip and negative (generating) for negative slip. The first term in Q_s represents the reactive power absorbed by the machine reactances. The angle δ_k depends heavily on slip S and R_r :

$$\delta_k = 0 \quad \text{for} \quad |S \omega_1 L_{sc}| \gg R_r \quad (1.61)$$

$$\delta_k = \frac{\pi}{2} \quad \text{for} \quad S = 0$$

$$0 < \delta_k < \frac{\pi}{2} \quad \text{for} \quad S > 0 \quad (1.62)$$

$$\frac{\pi}{2} < \delta_k < \pi \quad \text{for} \quad S < 0$$

To bring more generality to the P_s and Q_s dependences on δ , we represent P_s , Q_s as a function of $(\delta + \delta_k(S))$ (Figure 1.13). We may separate the two components in P_s and Q_s :

$$P_s = P_{ss} + P_{as} \quad (1.63)$$

$$Q_s = Q_{ss} + Q_{as} \quad (1.64)$$

P_{ss} and Q_{ss} are dependent on $(\delta + \delta_k(S))$, while P_{as} and Q_{as} are slip dependent only.

The variable $\delta + \delta_k(S)$ greatly simplifies the graphs, but care must be exercised when the actual power angle operation zone is computed. It is evident that for a voltage-fed rotor circuit — (V_r, δ) given — there is a certain difference between motor and generator operation zones, because the asynchronous power is positive (motoring) for $S > 0$ and negative (generating) for $S < 0$.

The sign of S does not influence reactive power Q_s ($\delta + \delta_k(S)$), but again, $\delta_k(S)$ depends on slip. To “produce” zero reactive power stator conditions, the rotor voltage ratio V_r/V_s has to be increased.

The peak active power is larger in motoring for $S > 0$ (subsynchronous) operation and, respectively, in generating for $S < 0$ (supersynchronous). Notice that WRIG peak stator active power is determined by the short-circuit ($\omega_1 L_{sc}$) rather than no-load ($\omega_1 L_s$) reactance. However, as $V_r/V_s \ll 1$, the peak active power is not very large, though larger than in SGs in general. The electromagnetic power ($R_s = 0$, $P_{Fe} = 0$) is as follows:

$$P_{elm} = P_s = T_e \frac{\omega_1}{p_1} \quad (1.65)$$

So, the electromagnetic torque is strictly proportional to stator active power P_s (for zero stator losses).

1.6.2 Rotor Power vs. Power Angle

The rotor electric active and reactive powers P_r^r , Q_r^r are as follows:

$$P_r^r + jQ_r^r = 3V_r \underline{I_r}^* \quad (1.66)$$

The rotor “produced” equivalent reactive power Q_r , seen from the stator (at stator frequency) is

$$Q_r = \text{Imag} \left(\frac{3V_r \underline{I_r}^*}{S} \right) \quad (1.67)$$

From Equation 1.55 and Equation 1.57,

$$P_r^r = \frac{3V_r^2 R_r}{R_r^2 + (S\omega_1 L_{sc})^2} + 3V_r V_s \frac{L_{lm}}{L_s} \frac{S \cdot \sin(\delta - \delta_k)}{\sqrt{R_r^2 + (S\omega_1 L_{sc})^2}} \quad (1.68)$$

*rotor copper losses synchronous rotor power
with shorted stator*

$$Q_r^r = \frac{3V_r^2 S\omega_1 L_{sc}}{R_r^2 + S\omega_1 L_{sc}^2} - 3V_r V_s \frac{L_{lm}}{L_s} \frac{S \cdot \cos(\delta - \delta_k)}{\sqrt{R_r^2 + (S\omega_1 L_{sc})^2}} \quad (1.69)$$

*reactive power absorbed synchronous reactive
with shorted stator rotor power*

Similar graphs $P_r'(\delta - \delta_k)$ and $Q_r'(\delta - \delta_k)$ may be drawn by using these expressions, but they are of a smaller practical use than P_s and Q_s . They are, however, important for designing the rotor-side static power converter and for determining the total rotor electric power delivery, or absorption, during subsynchronous or supersynchronous operation.

1.6.3 Operation at Zero Slip ($S = 0$)

At zero slip, from Equation 1.62, it follows first that $\delta_k = \pi/2$. Finally, from Equation 1.59 and Equation 1.60,

$$P_s = -3V_s V_r \frac{L_m}{R_r L_s} \sin\left(\delta + \frac{\pi}{2}\right) \quad (1.70)$$

$$Q_s = \frac{3V_s^2}{\omega_l L_s} - 3 \frac{V_s V_r L_m}{R_r L_s} \cos\left(\delta + \frac{\pi}{2}\right) \quad (1.71)$$

$$I_r = \frac{V_r}{R_r} \quad (1.72)$$

Note again that the rotor voltage is considered in stator coordinates. The power angle ($\delta + \frac{\pi}{2}$) is typical for SGs, where it is denoted by δ_u (the phase shift between rotor-induced emf and the phase voltage).

For operation at zero slip ($S = 0$), when the rotor circuit is DC fed, all the characteristics of SGs hold true. In fact, it seems adequate to run the WRIG at $S = 0$ when massive reactive power delivery (or absorption) is required. Though active and reactive power capability circles may be defined for WRIG, it seems to us that, due to decoupled fast active and reactive power control through the rotor-connected bidirectional power converters (Chapter 8, in *Synchronous Generators*), such graphs may become somewhat superfluous.

1.7 Autonomous Operation of WRIG

Insularization of WRIGs, in case of need, from the power grids, caused by excess power in the system or stability problems, leads to autonomous operation. Autonomous operation is characterized by the fact that voltage has to be controlled, together with stator frequency (at various rotor speeds in the interval $[1 \pm |S_{max}|]$), in order to remain constant under various active and reactive power loads. Whatever reactive power is needed by the consumers, it has to be provided from the rotor-side converter after covering the reactive power required to magnetize the machine. When large reactive power loads are handled, it seems that running at constant speed and zero slip ($S = 0$) would be adequate for taking full advantage of the rotor-side static converter limited ratings and for limiting rotor windings and converter losses. On the other hand, for large active loads, supersynchronous operation is suitable, as the WRIG may be controlled to operate around unity power factor while keeping the stator voltage within limits. Subsynchronous operation should be used when part loads are handled in order to provide for better efficiency of the prime mover for partial loads. The equivalent circuit (Figure 1.8) may easily be adapted to handle autonomous loads under steady state (Figure 1.14).

For autonomous operation, the stator voltage V_s is replaced by the following:

$$\underline{V}_s = -(R_{Load} + jX_{Load})I_s \quad (1.73)$$

In these conditions, retaining the power angle δ as a variable does not seem to be so important. The rotor voltage “sets the tone” and may be considered in the real axis: $\underline{V}_r = V_r$. Neglecting the stator

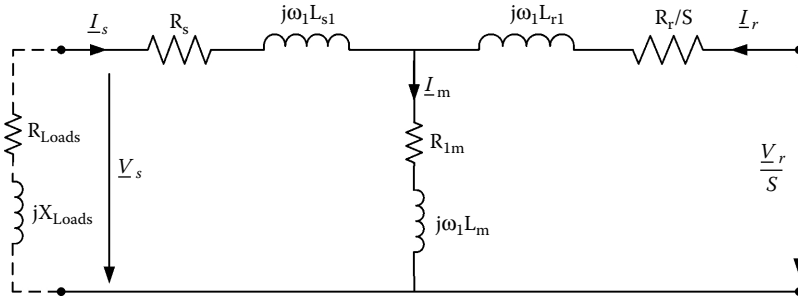


FIGURE 1.14 Equivalent circuit of wound rotor induction generator (WRIG) for autonomous operation.

resistance R_s does not bring any simplification, as it is seen in series with the load (Equation 1.73):

$$\begin{aligned} [R_s + R_{Load} + j(X_{Load} + X_{sl})]I_s &= -jX_{lm}(I_s + I_r) = E_m(I_m) \\ (R_s + jSX_{rl})I_r - V_r &= -jSX_{lm}(I_s + I_r) = E_m \cdot S; \quad I_m = I_s + I_r \end{aligned} \quad (1.74)$$

Both equations are written in stator coordinates (at frequency ω_1 for all reactances). We may consider now the WRIG as being supplied only from the rotor, with the stator connected to an external impedance. In other words, the WRIG becomes a typical induction generator fed through the rotor, having stator load impedance. It is expected that such a machine would be a motor for positive slip ($S > 0$, $\omega_r < \omega_1$) and a generator for negative slip ($S < 0$, $\omega_r > \omega_1$).

This is a drastic change of behavior with respect to the WRIG connected at a fixed frequency and voltage (strong) power grid, where motoring and generating are practical both subsynchronously and supersynchronously.

By properly adjusting the rotor frequency ω_2 with speed ω_r to keep ω_1 constant and controlling the amplitude and phase sequence of rotor voltage V_r , the stator voltage may be kept constant until a certain stator current limit, for given load power factor, is reached.

To obtain the active and reactive powers of the stator and the rotor P_s , Q_s , P_r , Q_r , solving first for the stator and rotor currents in Equation 1.74 is necessary. Neglecting the core loss resistance R_{1m} ($R_{1m} = 0$) yields the following:

$$\begin{aligned} I_s &= \frac{V_r}{R_{se}(S) + jX_{se}(S)} \\ R_{se}(S) &= \frac{R_r X_{s+l} - SX_r X_{s+l}}{X_{lm}} \\ X_{se}(S) &= \frac{SX_r R_{s+l} + X_{s+l} R_r}{X_{lm}} + SX_{lm} \\ R_{s+l} &= R_s + R_{loads}; \quad X_{s+l} = X_s + X_{loads} \\ X_s &= X_{sl} + X_{lm}; \quad X_r = X_{rl} + X_{lm} \end{aligned} \quad (1.75)$$

The active and reactive powers of stator and rotor are straightforward:

$$\begin{aligned} P_s &= 3I_s^2 R_{Loads} > 0 \quad S < 0 \text{ generating} \quad S > 0 \text{ motoring} \\ Q_s &= 3I_s^2 X_{Loads} < > 0 \end{aligned} \quad (1.76)$$

Also, $\underline{I}_r$ from Equation 1.74 is

$$\underline{I}_r = j \frac{(R_{s+l} + jX_{s+l})\underline{I}_s}{X_{lm}} \quad (1.77)$$

$$P_r^r + jQ_r^r = 3(V_r \cdot I_r^*) \quad (1.78)$$

The mechanical power P_m is simply

$$P_m = \frac{1-S}{S} [3R_r I_r^2 - P_r^r] \quad (1.79)$$

$$Q_r^r - 3(X_{sl} I_s^2 + X_{sl} I_r^2 + X_{lm} I_m^2) = Q_s \quad (1.80)$$

As the machine works as an induction machine fed to the rotor, with passive impedance in the stator, all characteristics of it may be used to describe its performance. The power balance for motoring and generating is described in Figure 1.15a and Figure 1.15b.

Note that subsynchronous operation as a motor is very useful when self-starting is required. The stator is short-circuited ($R_{load} = X_{load} = 0$), and the machine accelerates slowly (to observe the rotor-side converter P_{low} rating) until it reaches the synchronization zone $\omega_r(1 \pm |S_{max}|)$. Then the stator circuit is opened, but the induced voltage in the stator has a small frequency. Consequently, the phase sequence in the rotor voltages has to be reversed to obtain $\omega_1 > \omega_r$ for the same direction of rotation. This is the beginning of the resynchronization control mode when the machine is free-wheeling. Finally, within a few milliseconds, the stator voltage and frequency conditions are met, and the machine stator is reconnected to the load.

Induction motoring with a short-circuited stator is useful for limited motion during bearing inspections or repairs.

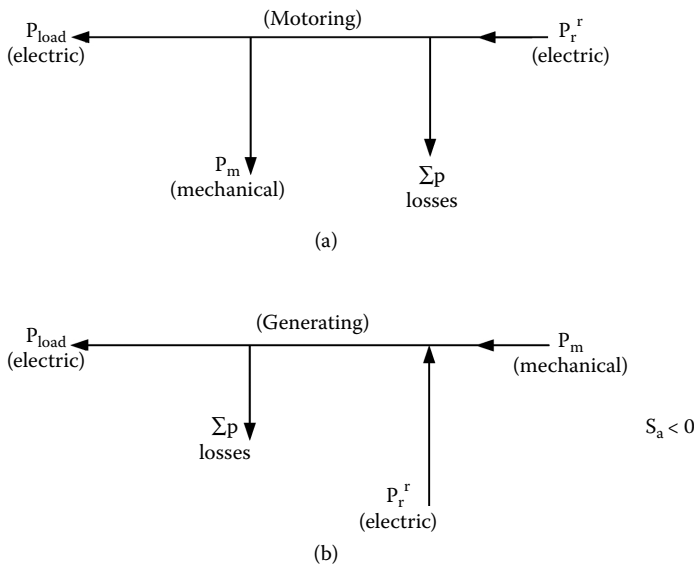


FIGURE 1.15 Power balance: (a) $S_a > 0$ and (b) $S_a < 0$.

Autonomous generating (on now-called ballast load) may be used as such and when, after load rejection, fast braking of the mover is required to avoid dangerous overspeeding until the speed governor takes over.

The stator voltage regulation in generating may be performed through changing the rotor voltage amplitude while the frequency ω_1 is controlled to stay dynamically constant by modifying frequency ω_2 in the rotor-side converter.

Example 1.2

For the WRIG in Example 1.1 at $S = -0.25$, $f_1 = 50$ Hz, $I_s = I_{sN}/2 = 602$ A, $V_r = V_{rmax} = V_s$, $\cos \varphi_s = 1$, compute the following:

- The load resistance R_{loads} per phase in the stator
- The load (stator voltage) V_s and load active power P_s
- The rotor current and active and reactive power in the rotor P_r , Q_r
- The no-load stator voltage for this case and the phasor diagram

After the computations are made, discuss the results.

Solution

- We have to go straight to Equation 1.75, with, $R_s = R_r = 0.018 \Omega$, $X_{sl} = X_{rl} = 0.018 \Omega$,
- $X_{lm} = 14.4 \Omega$, $X_s = X_r = 14.58 \Omega$, $V_{sphase} = 6000/\sqrt{3}$ V, $S = -0.25$, $I_s = 602$ A, $X_{load} = 0$ ($\cos \varphi_s = 1$), where the only unknown is R_{loads} :

$$R_{se}(S) = \left(\frac{0.018}{0.018} + R_{loads} \right) - \frac{(-0.25 \cdot 14.88 \cdot 14.88)}{14.4} = 3.69 + 1.25 \cdot 10^{-3} \cdot R_{loads}$$

$$X_{se}(S) = \left(-0.25 \cdot \frac{14.58}{0.018} + R_{loads} \right) - \frac{(14.58 \cdot 0.018)}{14.4} = -3.5938 - 0.253 \cdot R_{loads}$$

$$I_{-s} = 602 = \frac{V_r}{R_{se}(S) + jX_{se}(S)} = \frac{6000/\sqrt{3}}{(3.69 + 1.25 \cdot 10^{-3} R_{load}) - j(3.594 - 0.253 R_{load})}$$

Consequently, $R_{loads} \approx 3.276 \Omega$.

- The stator voltage per phase V_s is simply ($\cos \varphi_s = 1$).

$$(V_s)_{phase} = R_{load} I_s = 3.276 \cdot 602 = 1972 \text{ V}$$

$$P_s = -3 R_{loads} I_s^2 = -3 \cdot 3.276 \cdot 602^2 = -3.5617 \text{ MW}$$

- The rotor current (Equation 1.77) is

$$I_r = j \frac{(R_s + R_{loads} + jX_{s+l}) I_s}{X_{lm}}$$

with $I_s = 602 \cdot (0.641 + j0.767)$

So,

$$I_r = j \frac{(0.018 + 3.276 + j14.58)}{14.4} \cdot 602 \cdot (0.641 + j0.767) = 624.86 \angle +217^\circ$$

The active and reactive powers in the rotor are as follows:

$$P_r^r + jQ_r^r = 3V_r I_r^* = 3 \cdot \frac{6000}{\sqrt{3}} \cdot 624(-0.79 + j0.6018) = -5.116 \text{ MW} + j3.907 \text{ MVAR}$$

So, the rotor circuit absorbs reactive power to magnetize the machine, but it delivers active power, together with the stator. The mechanical power covers for all losses in the machine and produces both P_s and Q_s :

$$P_m - \Sigma p = |P_s + P_r| = -3.5617 - 5.116 = -8.6777 \text{ MW}$$

The losses considered in our example are only the winding losses:

$$\Sigma p = 3R_s I_s^2 + 3R_r I_r^2 = 3 \cdot 0.018 \cdot (602^2 + 624.86^2) = 40.654 \text{ KW}$$

So, the mechanical power is as follows:

$$P_m = 8.6717 + 0.04654 = 8.712 \text{ MW}$$

- The no-load voltage in the stator for the above conditions is simply

$$E_m = X_{lm} I_m; \quad \underline{I}_m = \underline{I}_s + \underline{I}_r$$

$$\underline{I}_m = 602 \angle 50.13^\circ + 624.86 \angle 217^\circ = -107.75 + j85.69$$

This is the magnetization current for the airgap flux:

$$E_m = X_{lm} I_m = 14.4 \cdot 137.669 = 1982.4 \text{ V}$$

The voltage regulation is very small:

$$\Delta V = \frac{E_m - V_s}{V_s} = \frac{1982.4 - 1972}{1982.2} = 0.5246\%$$

The current and voltage phasors are shown in [Figure 1.16](#).

Discussion

To force the delivery of notable active power from the machine, we considered $V_{rmax} = V_{sn} = 3.468 \text{ V}$; the trouble is that V_r is reduced to the stator, and thus, for our case, when the turns ratio is defined by $1/S_{max} = 4.0$, the actual rotor voltage meant by $V_r = V_s$ would, in fact, imply $V_r^r = 4.0 V_r = 4.6000/\sqrt{3} = 13.872 \text{ kV/phase}$.

In contrast, when the same machine (Example 1.1) delivered the power $P_s = 12.5 \text{ MW}$ through the stator (rated) plus 2.99 MW through the rotor at $S = -0.25$, $f_1 = 50 \text{ Hz}$, the rotor voltage V_r was only $V_r = 847 \text{ V}$. With the same rotor/stator turn ratio of 4.0, the actual rotor voltage would be, in this latter case, $V_r^r = 847 \cdot 4 = 3388 \text{ V}$, which is very close to the rated stator voltage, as intended from the start.

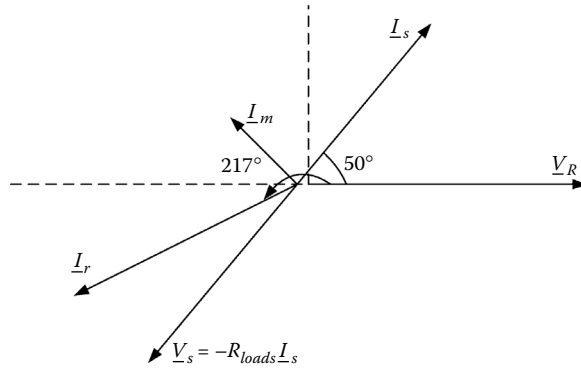


FIGURE 1.16 Phasors for autonomous wound rotor induction generator (WRIG) operation at $S = -0.25, f_1 = 50$ Hz.

To reduce the rotor output power and voltage V_r and increase the stator output, the slip has to be reduced drastically. Let us consider $I'_s = 700$ A and $V_r = 847$ V (as it was in Example 1.1) but for $S = -0.05$. Repeating the calculations as above, we obtain $R_{\text{loads}} \approx 5.734 \, \Omega$. The stator voltage V'_s is as follows:

$$V_s' = I_s' R_{load} = 700 \cdot 5.734 = 4.0 \text{ kV/phase}$$

The stator power $P_s = -3R_{loads} \cdot I_s'^2 = 3V_s' I_s' = 3 \cdot 4 \cdot 10^3 \cdot 700 = -8.40 \text{ MW}$.

The stator and rotor currents are thus,

$$\begin{aligned}\underline{I}_s &= \frac{V_r}{R_{sc}(s) + jX_{sc}(s)} = \frac{847}{0.7535 - j0.9887} \\ \underline{I}_r &= \underline{I}_s j \frac{(R_s + R_{load} + jX_s)}{X_{lm}} \\ &= 847 \cdot (0.606 + j0.795) \cdot j \cdot \frac{(0.018 + 5.734 + j14.4)}{14.4} = 911 \angle 200.4^\circ\end{aligned}$$

The rotor electric power is

$$P_r + jQ_r^r = 3V_r I_r^* = 3.847 \cdot 911 \angle 159.6 = -2.381 \text{ MW} + j0.806 \text{ MVAR}$$

Again, the reactive power in the rotor is absorbed by the machine for magnetization, while reasonable power is delivered by the rotor. The problem is that for $f_1 = 50$ Hz and $S = -0.05$, the speed we are talking about is $\omega_r = \omega_1(1-s) = 1.05\omega_1$.

Should the speed be large, say corresponding to $S = -0.25$, the power delivered at maximum rotor voltage (874 V when stator is reduced and $847 \times 4 = 3388$ V in reality), should be notably smaller than the value calculated; in fact, 16 times smaller.

The low reactive power required is due to the fact that the machine was designed with a high magnetization reactance (in P.U.; $x_{lm} = 5$), and the slip is now reasonable ($S = -0.05$).

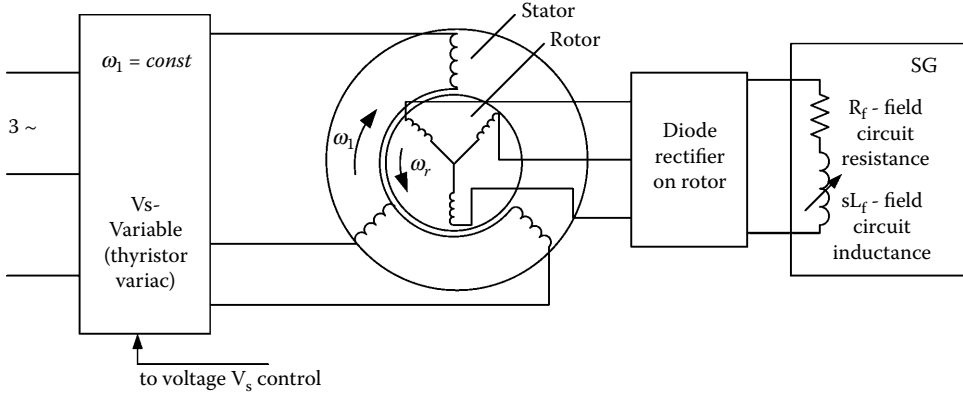


FIGURE 1.17 Wound rotor induction generator (WRIG) as brushless exciter.

1.8 Operation of WRIG in the Brushless Exciter Mode

With a brushless exciter, the power is delivered through the rotor, after rectification, to the excitation circuit of a synchronous generator (Figure 1.17). The commutation in the diode rectifier causes harmonics in the rotor current, but for its fundamental, the power factor may be considered as unity. The diode rectifier commutation causes some voltage reduction as already shown in Chapter 6 in *Synchronous Generators* (the paragraph on excitation systems).

The rotor rotates opposite to the stator mmf, and thus,

$$\omega_2 = \omega_1 + \omega_r > \omega_1 \quad (1.81)$$

The frequency in the rotor is at its minimum at zero speed and then increases with speed. If the WRIG is provided with a number of poles that is notably larger than that of the SG, then the frequency ω_2 would be larger than ω_1 :

$$\omega_2 = \omega_1 + 2\pi n_1 p_1; \quad n_1 = \frac{f_1}{p_g} \quad (1.82)$$

with p_g -pole pairs in the SG with excitation that is fed from the WRIG exciter:

$$\omega_2 = \omega_1 \left(1 + \frac{p_1}{p_g} \right) \quad (1.83)$$

The larger p_1/p_g , the higher the rotor (slip) frequency; with $p_1/p_g = 3, 4$, good results may be obtained. The WRIG-exciter is supplied through a static variac at constant frequency ω_1 , so the converter's cost is low.

The machine equations (Equation 1.42) remain valid, but we will use ω_2 instead of $S\omega_1$:

$$\begin{aligned} \underline{I}_s R_s - \underline{V}_s &= -j\omega_1 \underline{\Psi}_s = -j\omega_1 (L_s \underline{I}_s + L_{lm} \underline{I}_r) \\ \underline{I}_r R_r + \underline{V}_r &= -j\omega_2 \underline{\Psi}_r = -j\omega_2 (L_r \underline{I}_r + L_{lm} \underline{I}_s) \end{aligned} \quad (1.84)$$

The speed ω_r is now negative ($\omega_r < 0$), that is, $\omega_1 > 0$ and $\omega_2 > 0$. The slip $S = \omega_2/\omega_1 > 1$.

We already used the positive sign (+) on the left side of rotor equation to have positive power for generating. Also, for simplicity, a resistance load will be considered:

$$\underline{V}_r = \underline{I}_r \cdot R_{loadr} \quad (1.85)$$

The Equation 1.84 with Equation 1.85 may be solved simply for stator and rotor currents:

$$\underline{I}_s = \frac{j\underline{I}_r(R_r + j\omega_2 L_r + R_{loadr})}{\omega_2 L_{lm}} \quad (1.86)$$

$$\underline{I}_r = \frac{\underline{V}_s}{j \frac{(R_s + R_{load} + j\omega_2 L_r)(R_s + j\omega_1 L_s)}{\omega_2 L_{lm}} + j\omega_1 L_m} \quad (1.87)$$

The electromagnetic torque T_e is

$$T_e = 3p_1 \operatorname{Re} al \left(j \underline{\Psi}_s \underline{I}_s^* \right) = 3p_1 L_{lm} \operatorname{Re} al \left(j \underline{I}_s \underline{I}_r^* \right) \quad (1.88)$$

At zero speed, the WRIG-exciter works as a transformer, and all the active and reactive power is delivered by the stator. When the speed increases — with resistive load in the rotor circuit — the stator “delivers” the reactive power to magnetize the machine and the active power to cover the losses and some part of the load active power.

The bulk of the active power to the load comes, however, from the mechanical power P_m . The higher the ratio ω_2/ω_1 , the higher the P_m contribution to P_r (rotor-delivered active power).

Example 1.3: WRIG as Brushless Exciter

Consider a WRIG with the main data: $R_s = R_r = 0.015 \text{ P.U.}$, $L_{sl} = L_{rl} = 0.14 \text{ P.U.}$, $L_{lm} = 3 \text{ P.U.}$, $V_{SNI} = 440 \text{ V}(star)$, $I_{SN} = 1000 \text{ A}$, the frequency $f_1 = 60 \text{ Hz}$, and the rotor speed $n_N = 1800 \text{ rpm}$. The number of pole pairs is $p_1 = 6$. The rotor-to-stator turns ratio is $a_{rs} = 1$. Determine the following:

- The rotor frequency $f_2(\omega_2)$ and the ideal maximum no-load rotor voltage
- The rotor-side load resistance voltage, current, power P_{r0} at zero speed, and $I_r = 1000 \text{ A}$ in the rotor
- The required stator voltage, current, and input active and reactive powers P_s , Q_s , for the same load resistance R_{load} and current load $I_r = 1000 \text{ A}$, but at $n_N = 1800 \text{ rpm}$

Solution

- The rotor-side frequency $f_2(\omega_2)$ is simply as follows (Equation 1.82):

$$\omega_2 = \omega_1 + \frac{2\pi n_1 p_1}{\omega_1} = \left(1 + \frac{2\pi \cdot \frac{1800}{60} \cdot 6}{2\pi 60} \right) = 4\omega_1$$

So, $f_2 = 4f_1 = 240 \text{ Hz}$.

The ideal no-load rotor voltage V_r^r (unreduced to the stator, for full stator voltage at speed n_N), is as follows:

$$V_{r0}^r = a_{rs} \cdot V_s \cdot \frac{\omega_2}{\omega_1} = 1 \cdot 440 \cdot \frac{4}{1} = 1760 \text{ V (line voltage, RMS)}$$

The rotor circuit might be designed to comply with this voltage during an excitation 4/1 forcing. At zero speed ($\omega_2 = \omega_1$), the ideal rotor voltage would be

$$(V_{r0})_{\text{stall}} = a_{rs} \cdot V_s \cdot \frac{\omega_1}{\omega_1} = 1 \cdot 440 \cdot \frac{1}{1} = 440 \text{ V}$$

- The machine parameters in Ω (all reduced to the stator) are as follows:

$$X_n = \frac{V_{SN}/\sqrt{3}}{I_{SN}} = \frac{(440/\sqrt{3})}{1000} = 0.2543 \text{ } \Omega$$

So,

$$R_s = R_r = (R_s)_{P.U.} \cdot X_n = 0.015 \cdot 0.2543 = 3.8145 \cdot 10^{-3} \text{ } \Omega$$

$$L_{sl} = L_{rl} = (X_{sl})_{P.U.} \cdot \frac{X_n}{\omega_1} = 0.14 \cdot \frac{0.2543}{2\pi 60} = 9.45 \cdot 10^{-5} \text{ H}$$

$$L_{lm} = (X_{lm})_{P.U.} \cdot \frac{X_n}{\omega_1} = 3 \cdot \frac{0.2543}{2\pi 60} = 2.0247 \cdot 10^{-3} \text{ H}$$

At zero speed, $\omega_2 = \omega_1$, the rotor current I_r may be calculated from the following (Equation 1.87):

$$\begin{aligned} (I_r)_{\omega_2=\omega_1} &= \frac{jV_s}{\frac{(R_s + R_{load} + j\omega_2 L_r)(R_s + j\omega_1 L_s)}{\omega_2 L_{lm}} + j\omega_1 L_{lm}} \\ &= \frac{(440/\sqrt{3})}{\frac{(3.8145 \cdot 10^{-3} + R_{load} + j2\pi \cdot 60 \cdot 2.119 \cdot 10^{-3})(3.8145 \cdot 10^{-3} + j2\pi \cdot 60 \cdot 2.119 \cdot 10^{-3})}{2\pi \cdot 60 \cdot 2.0247 \cdot 10^{-3}} + j2\pi \cdot 60 \cdot 2.0247 \cdot 10^{-3}} \end{aligned}$$

Finally,

$$R_{load} = 0.226 \text{ } \Omega$$

So, the rotor voltage V_r (reduced to the stator) is

$$V_r = R_{load} \cdot I_r = 0.226 \cdot 1000 = 226 \text{ V}$$

For voltage regulation,

$$\Delta V = \frac{V_s - V_r}{V_s} = \frac{254 - 226}{254} = 0.1102 = 11.02\%$$

The large leakage reactances of the stator and the rotor are responsible for this notable voltage drop (notable for a transformer or an induction machine, but small for an SG of any type).

The rotor-delivered power P_r is as follows:

$$P_r = 3V_r \cdot I_r = 3 \times 220 \cdot 1000 = 678 \text{ KW}$$

- Now, we make use of Equation 1.87 to calculate the stator voltage required for $I_r = 1000$ A, with

$$\begin{aligned}\underline{V}_s &= \underline{I}_r j \left[\frac{(R_r + R_{loadr} + j\omega_2 L_r) \cdot (R_s + j\omega_1 L_{lm})}{\omega_2 L_{lm}} \right] \\ &= 1000 j \left[\frac{(0.0038145 + 0.226 + j \cdot 2\pi \cdot 240 \cdot 2.119 \cdot 10^{-3})(0.0038145 + j \cdot 2\pi \cdot 60 \cdot 2.119 \cdot 10^{-3})}{2\pi \cdot 240 \cdot 2.0247 \cdot 10^{-3}} \right] \\ &\quad + 2\pi \cdot 60 \cdot 2.204 \cdot 10^{-3} \\ \underline{V}_s &= 1000 j \cdot (-0.0721 + j0.06406) = -64.06 - j72.1 \\ V_s &= 96.8 \text{ V (RMS per phase)}\end{aligned}$$

The stator $\underline{I}_s$ from Equation 1.86 is

$$\underline{I}_s = j \underline{I}_r \frac{(R_r + j\omega_2 L_r + R_{load})}{\omega_2 L_{lm}} = j \frac{1000 \cdot (0.02298 + j3.1938)}{3.0516} = -1046.6 + j75.3; \quad I_s = 1049.3 \text{ A} > I_r$$

The stator active and reactive powers are as follows:

$$\begin{aligned}P_s + jQ_s &= 3 \underline{V}_s \underline{I}_s^* = 3 \cdot (-64.06 - j72.1) \cdot (-1046.6 - j75.3) \\ P_s &= 184.752 \text{ KW}, \quad Q_s = 240.751 \text{ KVAR}\end{aligned}$$

The delivered electric power through the rotor P_r is still 678 kW, as the load resistance and current were kept the same, but most of the power now comes from the shaft as $P_s \ll P_r$.

A few remarks are in order:

- As the machine is rotated, less active power is delivered through the stator, with much of it “extracted” from the shaft (mechanically). This is a special advantage of this configuration.
- With the machine in motion ($\omega_r = 3\omega_1$), the required stator voltage decreases notably. A static variac may be used to handle such a 1/5 voltage reduction easily.
- The machine magnetization is provided by the stator, and because $\omega_2 = 4\omega_1$, the power factor in the stator is poor.
- The magnetization by the stator is also illustrated by $I_s > I_r$.
- The stator voltage reserve at full speed may be used for forcing the excitation (load) current in the supplied synchronous machine excitation, but, in that case, the rotor voltage would increase above the rated value (440 V root mean squared [RMS]/line). The rotor winding insulation and the flying diode rectifier have to be sized for such events.
- The capability of the WRIG to serve as an exciter from zero speed — demonstrated in this example — makes it a good solution when the excitation power is required from zero speed, as is the case in variable-speed large synchronous motors or generators.
- The internal reactance of the WRIG is important to know in order to assess voltage regulation and to model the machine with rectified output.
- To emphasize the “synchronous” reactance of WRIG as an exciter, the stator current is eliminated from the stator equation by introducing the stator flux $\underline{\Psi}_s$:

$$\underline{\Psi}_r = \frac{L_m}{L_s} \underline{\Psi}_s + L_{sc} \underline{I}_r \quad (1.89)$$

The rotor equation (Equation 1.84) may be written now as follows:

$$\underline{V}_r = -j\omega_2 \frac{L_m}{L_s} \underline{\Psi}_s - (R_r + j\omega_2 L_{sc}) \underline{I}_r = \underline{E}_r - \underline{Z}_{ex} \underline{I}_r \quad (1.90)$$

$$\underline{Z}_{ex} = R_r + j\omega_2 L_{sc} \quad (1.91)$$

The term $\underline{Z}_{ex}$ represents the internal (synchronous) impedance of WRIG as an exciter source. The first term in Equation 1.90 is the emf $\underline{E}_r^*$:

$$\underline{E}_r = -j\omega_2 \frac{L_m}{L_s} \underline{\Psi}_s \quad (1.92)$$

The stator flux may be considered variable, with stator voltage as follows ($R_s = 0$):

$$-j\omega_1 \underline{\Psi}_s \approx \underline{V}_s \quad (1.93)$$

Consequently,

$$\underline{E}_r = \frac{V_s L_m \omega_2}{L_s \omega_1} \quad (1.94)$$

An equivalent circuit based on Equation 1.90 and Equation 1.94 may be built (Figure 1.18).

Basically, the emf $\underline{E}_r'$ varies with ω_2 (that is, with speed for constant ω_1) and with the stator voltage V_s . The “synchronous” reactance of the machine is, in fact, the short-circuit reactance.

So, the voltage regulation is reasonably small, and the transient response is expected to be swift; a definite asset for excitation control.

As the frequency in the rotor is large ($\omega_2 > \omega_1$), the core losses in the machine have to be considered. One way to do this is to “hang” a core resistance R_{Fe} in parallel with the emf E_r' , R_{Fe} may be taken as a constant, to be determined either from measured or calculated core losses P_{Fe} :

$$P_{Fe} = 3 \frac{E_r'^2}{R_{Fe}} \quad (1.95)$$

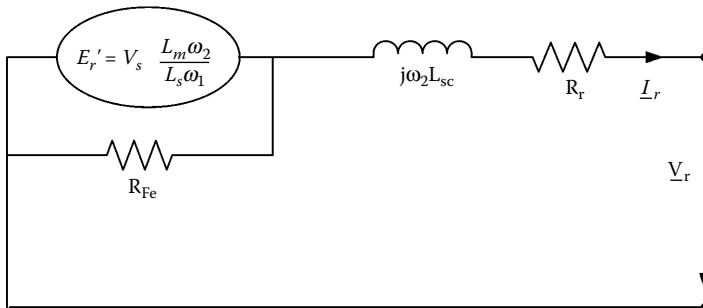


FIGURE 1.18 Equivalent circuit (phase) for wound rotor induction generator (WRIG) as an exciter source.

1.9 Losses and Efficiency of WRIG

The loss components in WRIG may be classified as follows:

- Stator-winding losses
- Stator core losses
- Rotor-winding losses
- Mechanical losses

The stator-winding losses are due to alternative currents flowing into the stator windings. With constant frequency ($f_1 = 50$ (60) Hz), only in medium and large power machines is the skin effect important. Roebel bars may be used in large power WRIGs to keep the influence of the skin effect coefficient below 0.33 (that is, 33% additional losses):

$$p_{\cos} = 3I_s^2(R_s)_{dc} \cdot (1 + K_{skin}^s) \quad (1.96)$$

In the rotor, the frequency $f_2 = Sf_1$, and with WRIGs, $|f_2| < 0.3f_1$. The rotor-to-stator turn ratio a_{rs} is chosen to be larger than $1(a_{rs} = 1/|S_{max}|)$ for low stator voltage WRIGs (up to 2 to 3 MW) and, in this case, the skin effect in the rotor is negligible.

However, in large machines, as the rotor voltage will probably not go over 4 to 6 kV (line voltage), even in the presence of specially built slip-rings, the rotor currents are large, in the range of thousands of amperes, again, transposed conductors are needed for the rotor windings. There will be some skin effect, but, as the rotor frequency $|f_1| < 1/3f_1$, in general, its influence will be less important than in the stator ($K_{skin}^r < K_{skin}^s < 0.3$):

$$p_{cor} = 3I_{r1}^2(R_r)_{dc} \cdot (1 + K_{skin}^r) \quad (1.97)$$

For details on skin effect, see Chapter 7 in *Synchronous Generators*.

The fundamental stator core and rotor core losses may be approximated by an aggregated core-loss resistance R_{Fe} :

$$R_{Fe} = R_{Fes}(\omega_1) + R_{Fer}(\omega_2) \quad (1.98)$$

This is exposed to the airgap emf E_m :

$$\underline{E}_m = -jX_m I_m; \quad \underline{I}_m = \underline{I}_s + \underline{I}_r \quad (1.99)$$

So,

$$p_{Fes} = \frac{3(X_m I_m)^2}{R_{Fe}(\omega_1)} \quad (1.100)$$

$$p_{Fer} = \frac{3(X_m I_m)^2}{R_{Fer}(\omega_2)} \quad (1.101)$$

The values of stator and rotor core loss resistances R_{Fe} and R_{Fer} may be obtained through experiments or from the design process.

When $|\omega_2| < \omega_1$, the rotor core losses are definitely smaller than in the stator. This is not so when the WRIG is used as an exciter ($|\omega_2| \gg \omega_1$), and thus, even though the rotor core volume is larger in the stator, the rotor core losses are larger.

Additional losses occur in the stator and rotor windings in relation to the circuit time harmonics due (mainly) to the static power converter connected to the rotor. They are strongly dependent on the PWM strategy and on the switching frequency.

Additional core losses occur due to space and time harmonics in the mmf of stator and rotor windings, in the presence of double slotting. Current time harmonics bring additional core losses.

The additional space harmonics core losses occur on the rotor and stator surface toward the airgap. Generally, only the first slot harmonics $v_{ss} = \frac{N_s}{p_1} \pm 1$, $v_{rs} = \frac{N_r}{p_1} \pm 1$, as influenced by the corresponding first-order airgap magnetic conductance harmonics, are considered to produce surface core losses that deserve attention [2]. Current time harmonics, on the other hand, produce additional core losses mainly along a thin layer along the slot walls.

Mechanical losses include ventilator (if any) losses, bearing-friction losses, brush-friction losses, and windage losses (P_{mec}):

$$\eta = \frac{P_s + P_r}{P_m} = \frac{P_s + P_r}{P_s + P_r + \Sigma p} \quad (1.102)$$

$$\Sigma p = p_{cos} + p_{cor} + p_{Fe} + p_s + p_{mec} + p_{sr}$$

For generating, P_s , in Equation 1.102, is always considered positive (delivered), while P_r is positive (delivered) for supersynchronous operation, and $P_r < 0$ (absorbed) for subsynchronous operation. P_m is the mechanical (input) power. The slip-ring losses are denoted by p_{sr} and the strayload losses are denoted by p_s .

For details on efficiency (through iso-efficiency curves), see Reference [9].

1.10 Summary

- WRIGs are provided with three-phase AC windings on the rotor and on the stator. WRIGs are also referred to as DFIGs or DOIGs.
- WRIGs are capable of producing constant frequency (f_1) and voltage stator output power at variable speed if the rotor windings are controlled at variable frequency (f_2) and variable voltage. The rotor frequency f_2 is determined solely by speed n (rps) and $f_1/f_2 = f_1 - np_1$; p_1 equals the number of pole pairs; and p_1 is the same in the stator and in the rotor windings.
- WRIGs may operate as both motors and generators subsynchronously ($n < f_1/p_1$) and supersynchronously ($n > f_1/p_1$), provided the static power converter that supplies the rotor winding is capable of bidirectional power flow.
- The slip is defined as $S = \omega_2/\omega_1 = f_2/f_1$ and is positive for subsynchronous operation and negative for supersynchronous operation, and so is f_2 . Negative f_2 means the opposite sequence of phases in the rotor is followed.
- WRIG is adequate in applications with limited speed control range ($|S_{max}| < 0.2$ to 0.3), as the rating of the rotor-side static converter is around $P_{SN}|S_{max}|$, where P_{SN} is the rated stator power. The electric power P_r in the rotor is delivered for generating in supersynchronous operation and is absorbed in subsynchronous operation: $P_r \approx P_{SN}^* S$. The total maximum power P_t delivered supersynchronously is thus,

$$P_t = P_s + P_r = P_{SN} (1 + |S_{max}|)$$

- Consequently, in supersynchronous operation, the WRIG can produce significantly more total electric power than the rated power at synchronous speed ($S = 0$).
- WRIG may also operate at synchronism, as a standard synchronous machine, provided the rotor-side static power converter is able to handle DC power. Back-to-back voltage source PWM converters are adequate for the scope. It may be argued that, in this case ($S = 0$), a WRIG acts like a damperless SG. True, but this apparent disadvantage is compensated for by the presence of fast close-loop control of active and reactive power, which produces the necessary damping any time the machine deviates from synchronism. WRIG is also adequate to work as a synchronous condensator and contribute massively, when needed, to voltage control and stability in the power grid.

- WRIG has laminated iron cores with uniform slots to host the AC windings. Integer q (slots/pole/phase) windings are used. Open slots may be used only on one side of the airgap. Axial cooling unistack cores are now in use up to 2 to 3 MW, while axial–radial cooling multistack cores are necessary above 3 MW.
- To avoid parasitic synchronous torque, it suffices to have different numbers of slots in the rotor and the stator. With large q and chorded coils, the main mmf harmonics are reduced and also reduced are their asynchronous parasitic torques.
- The rotor-to-stator turn ratio a_{rs} may be chosen as unity, but in this case, a voltage matching transformer is needed between the static converter in the rotor and the local power grid. Alternatively, $a_{rs} = 1/|S_{max}| > 1$ when the transformer is eliminated.
- A WRIG may be magnetized either from the stator or from the rotor, so the magnetization curve may be calculated (or measured) from both sides.
- When the reactive power is delivered through the rotor (overexcitation), the stator may operate at the unity power factor. The lagging power factor in the stator seems to be a moderate to large burden on the rotor-side static converter kilovoltampere rating.
- A minimum kilovoltampere rating of the rotor-side static power converter is obtained when the stator power factor is leading (underexcitation $\Psi_r < \Psi_s$). Ψ_r and Ψ_s are, respectively, the rotor and the stator flux linkage amplitudes per phase.
- For operation at the power grid, synchronization is required. However, synchronization is much faster and easier than with SGs, because it may be performed at any speed $\omega_r > \omega_1(1 - |S_{max}|)$ by controlling the rotor-side converter in the synchronization mode to make the power grid and stator voltages of the WRIG equal to each other and in phase. The whole synchronization process is short, as the rotor voltage and frequency (phase) are controlled quickly by the static power converter without any special intervention by the prime mover's governor.
- The values of active power and reactive powers P_s , Q_s , P_r , Q_r vs. the rotor voltage (power) angle δ are somewhat similar to those in the case of cylindrical rotor SGs, but additional asynchronous power terms are present, and the stable operation zones depend heavily on the value and sign of slip (Figure 1.12). However, the decoupled active and reactive power control (see Chapter 2) eliminates such inconveniences to a great extent.
- The peak value of synchronous power components in P_s , P_r , for constant rotor flux, depend on the short-circuit reactance (impedance) of the machine. Voltage regulation is moderate, for the same reason.
- The reactive power Q_r^r , absorbed from the rotor-side converter at $f_2 = S f_1$ frequency, is “magnified” in the machine to the frequency f_1 , $Q_r = Q_r^r/|S|$, as it has to produce the magnetic energy stored in the short-circuit and magnetization inductances. Operation at unity power factor in the stator at full power leads, thus, to a moderate increase in rotor-side static converter kilovoltampere rating for $|S_{max}| < 0.25$.
- The WRIG may also operate as a stand-alone generator. It was demonstrated that such an operation is preferred for low reactive power requirements at low negative slips. Constant frequency, constant voltage output in the stator with autonomous load does not seem to be advantageous when the speed varies by more than $\pm 5\%$. Ballast loads may be handled at any speed effectively, at smaller slip.
- With the stator short-circuited, the WRIG may be run as a motor to start the prime mover, say, for pumping in a pump-storage plant.
- After acceleration to $\omega_r > \omega_1(1 - |S_{max}|)$, the stator circuit is opened, the sequence of rotor voltages is changed, and their frequency f_2 and amplitude are reduced to produce the conditions necessary for quick stator synchronization. After that, motoring or generating operation is commanded subsynchronously or supersynchronously.
- The WRIG may operate in the brushless exciter mode to produce DC power on the rotor side with a diode rectifier and thus feed the excitation of a synchronous machine from zero speed up to the desired speed.
- The stator is supplied through a static voltage changer (soft starter type) at constant frequency ω_1 , while the rotor moves such that the rotor frequency $\omega_2 = \omega_1 + |\omega_r| > \omega_1$. With $\frac{\omega_2}{\omega_1} = 3, 4$, good

performance is obtained. In all situations, the magnetization (reactive power) is delivered through the stator, but most of the load active power comes from the shaft mechanical power, and only a small part comes from the stator. At zero speed, however, all the excitation power is delivered by the stator, electrically.

- When the speed increases, for constant rotor voltage, the stator voltage of the WRIG exciter is reduced considerably. So, there is room for excitation forcing needs in the SG, provided the WRIG exciter insulation can handle the voltage. The internal impedance of WRIG for brushless exciter mode is, again, the short-circuit impedance. Thus, the commutation of diode reduction of the DC output voltage should be moderate.
- Besides fundamental winding and core losses, additional losses occur in the windings and magnetic cores of WRIGs due to space and time harmonics.
- The WRIG was proven to be reliable for delivering power at variable speed with very fast decoupled active and reactive control in industry up to 400 MW/unit. It is yet to be seen if the WRIG will get a large share in the electric power generation of the future, at low, medium, and high powers per unit.

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